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PHYSICS OF PLASMAS 20, 022903 (2013)

Ponderomotive force in the presence of electric fields

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This paper presents averaged equations of particle motion in an electromagnetic wave of arbitrary frequency with its wave vector directed along the ambient magnetic field. The particle is also subjected to an $\vec{E}\times\vec{B}$ drift and a background electric field slowly changing in space and acting along the magnetic field line. The fields, wave amplitude, and the wave vector depend on the coordinate along the magnetic field line. The derivations of the ponderomotive forces are done by assuming that the drift velocity in the ambient magnetic field is comparable to the particle velocity. Such a scenario leads to new ponderomotive forces, dependent on the wave magnetic field intensity, and, as a result, to the additional energy exchange between the wave and the plasma particles. It is found that the parallel electric field can lead to the change of the particle-wave energy exchange rate comparable to that produced by the previously discussed ponderomotive forces. [http://dx.doi.org/10.1063/1.4789874]

I. INTRODUCTION

As it is well known, the ponderomotive forces represent time-averaged nonlinear electromagnetic forces acting on the plasma in the presence of oscillating electromagnetic fields. The ponderomotive forces have become widely used in space plasma physics since they allow simplification of the dynamical plasma equations. Ponderomotive forces can affect the plasma density distribution and cause particle acceleration and energization. The dependence of this force on the wave amplitude and ambient magnetic field inhomogeneity, wave non-stationarity, specific wave mode, and some other plasma and wave parameters has been analyzed in a comprehensive review by Lundin and Guglielmi.[1] Particle finite Larmor radius effects and the effects of ion gyroharmonics in the ponderomotive force expression were considered,[2] as well as non-local temperature dependent effects.[3] All ponderomotive forces are characterized by a quadratic dependence on the amplitude of the electric field and their explicit formulation depends on the specific nonlinearity. The ponderomotive forces are usually divided into various categories, in particular, distinguished by their dependence on temporal or spatial wave intensity derivatives, e.g., see Lundin and Guglielmi.[1] Another type of ponderomotive force as discussed by Lundin and Guglielmi[1] is the Barlow force. It arises due to particle collisions and is independent of the wave intensity derivatives. Khazanov *et al.*[4] introduced an additional ponderomotive force, caused by the combination of the electromagnetic wave and particle gravitational acceleration along the ambient magnetic field, for the case of a homogeneous plasma. This force is also independent of the wave intensity derivatives.

Particles can be accelerated also by a parallel electric field, and subjected to perpendicular electric fields leading to drift velocities comparable to their thermal velocity, for example, in the Earth's magnetospheric auroral and polar zones. It is also the region of strong wave activity and therefore the ponderomotive force in this region can be essential for density cavity formation[5] and ion outflow.[6]

Ponderomotive forces are usually considered assuming that the particle drift velocities due to a slowly changing background electric field and a wave electric field are small compared to the particle velocity. These assumptions, however, are not always appropriate, and the general problem, without these restrictions, was considered by Milant'ev and Ogunniii.[7] Their resulting equations, however, are immediately applicable only for the homogeneous and stationary background magnetic field.

In this paper, we expand on the ponderomotive force studies of Khazanov *et al.*[4] and Khazanov *et al.*[8] by assuming that the drift velocity in the ambient magnetic field is comparable to the particle velocity. We also include a background electric field slowly changing in space and acting along the magnetic field line. It can be expected, in accordance with Khazanov *et al.*,[4] that this field initiates an additional ponderomotive force in the presence of the wave activity.

II. BASIC EQUATIONS

We assume that the background magnetic field as well as all other quantities in the problem are dependent only on the coordinate along the magnetic field line, s. Let us consider a particle immersed in ambient magnetic, $\vec{B}_0(s)$, and electric, $\vec{E}^c(s)$ fields, slowly changing along the magnetic field line, in the presence of an electromagnetic wave with angular frequency ω, the wave vector $\vec{k}(s)$ directed along $\vec{B}_0(s)$, and electric field given by

$$\vec{E}^t(s,t)=\vec{E}(s)\exp\left(i\int k(s)ds-i\omega t\right)+c.c.=\vec{E}+\vec{E}^*, \quad (1)$$

propagating along the ambient magnetic field, where $\vec{E}^*$ represent the complex conjugate. In an orthogonal coordinate system with unit vectors $\vec{e}_z=\frac{\vec{B}_0(s)}{B_0(s)}$, and $\vec{e}_x$ along the principal normal of the external magnetic field line, the expression for the magnetic field of the wave can be found from Maxwell's equations, using Eq. (1),

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$$\vec{B}^{t}(s,t)=\frac{c}{\omega}\left[\vec{k}\times\vec{E}+i\left(\vec{e}_{x}E_{y,s}-\vec{e}_{y}B_{0}\left(\frac{E_{x}}{B_{0}}\right)_{s}\right)\right.$$
$$\left.\times\exp(-i\vartheta)\right]+c.c.=\vec{B}^{(0)}+\vec{B}^{(1)}+c.c.=\vec{B}+\vec{B}^{*}, \tag{2}$$

where

$$\vartheta=\omega t-\int k(s)ds,\ \vec{B}^{(0)}=\frac{c}{\omega}\vec{k}\times\vec{E},$$

c is the velocity of light, and the subscript s denotes a partial derivative with respect to s, e.g., $a_s\equiv(a)_s\equiv\frac{\partial a}{\partial s}$.

In this paper, we consider slow changing electric fields $E^c_\perp$ and the wave electric field $E_\perp$ to be strong, and the drift velocities are of the same order as the particle velocity,

$$c\frac{E^c_\perp}{B_0}\sim \mathrm{v},\quad c\frac{E_\perp}{B_0}\sim \mathrm{v}, \tag{3}$$

while

$$E^c_{||}\ll E^c_\perp\quad\text{and}\quad E_{||}\ll E_\perp, \tag{4}$$

as it is required by the condition of drift approximation applicability. Parallel and perpendicular refers to the directions relative to the ambient magnetic field $\vec{B}_0(s)$. Our task is to average the equations of particle motion over fast cyclotron oscillations and oscillations in the wave field. The details of the method of averaging for a bi-periodical system of differential equations can be found in Khazanov *et al.*[8] and Morozov and Solov'ev.[9] To transform the equation of particle motion to a form appropriate for averaging, the particle velocity in cylindrical coordinates can be presented as

$$\vec{u}=\vec{\mathrm{v}}_{||}+\vec{\mathrm{v}}^c_d+\vec{\mathrm{v}}^t_d+\mathrm{v}_\perp(\vec{e}_x\cos\varphi+\vec{e}_y\sin\varphi). \tag{5}$$

The drift velocities across the ambient magnetic field due to the slowly changing convective electric field and wave electric field Eq. (1) are, respectively,

$$\vec{\mathrm{v}}^c_d=c\frac{\vec{E}^c_\perp\times\vec{B}_0}{B_0^2},$$
$$\vec{\mathrm{v}}^t_d=\frac{\varpi}{\omega}\frac{\vec{E}_\perp\times\vec{\Omega}-i\varpi\vec{E}_\perp}{\Omega^2-\varpi^2}+c.c.=\vec{\mathrm{v}}_d+\vec{\mathrm{v}}^*_d;\quad \varpi=\omega-k\mathrm{v}_{||};$$
$$\vec{\Omega}=\frac{\vec{B}_0}{c};\quad e=m=1. \tag{6}$$

As a result, taking into account Eqs. (1), (2), and (6), the equations of motion can be presented as

$$\begin{aligned}&\frac{d\vec{r}}{dt}=\vec{u}\\&\frac{d\mathrm{v}_{||}}{dt}=S_0+\left\{\rho\mathrm{v}_\perp\left(\mathrm{v}^c_{dx}e^{i\varphi}+\frac{\mathrm{v}_\perp}{4}e^{i2\varphi}\right)+\mathrm{v}_\perp e^{i\varphi}\right.\\&\quad\times\left[\left(\rho\mathrm{v}_{dx}+i\frac{B^-}{2c}\right)e^{-i\vartheta}+\left(\rho\mathrm{v}^*_{dx}+i\frac{B^{+*}}{2c}\right)e^{i\vartheta}\right]\\&\quad\left.+S+c.c\right\}.\end{aligned} \tag{7}$$

Here, $\rho=-\frac{1}{B_0}\frac{dB_0}{ds}$ is the curvature, $R^\pm=R_x\pm iR_y$ denotes the combinations of components of some arbitrary vector $\vec{R}$, and

$$S_0=E^c_{||}+\rho\left(\mathrm{v}^{c2}_{dx}+\mathrm{v}^{t2}_{dx}+\frac{\mathrm{v}^2_\perp}{2}\right)+\frac{\vec{e}_z}{c}\vec{\mathrm{v}}^t_d\times\vec{B}^t \tag{8}$$

$$S=E_{||}+\frac{\vec{e}_z}{c}\vec{\mathrm{v}}^c_d\times\vec{B}+2\rho\mathrm{v}^c_{dx}\mathrm{v}_{dx}. \tag{9}$$

In Eq. (8), the last term is proportional to ρ, which defines the characteristic scale of inhomogeneity $L\sim\frac{1}{\rho}$. Because $E^c_{||}\ll E^c_\perp$, (4), we can also assume that the term $\frac{\mathrm{v}^2_\perp}{2}$ (particle perpendicular kinetic energy) is not smaller than $E^c_{||}L$, and so, $S_0\sim\rho$.

We also will restrict the discussion below to the case of $S\sim\rho$. Note that the first two terms in Eq. (9) present the parallel electric field in a reference frame moving with the velocity $\vec{\mathrm{v}}^c_d$ and, for transverse waves generated in a plasma drifting with this velocity, their sum becomes zero. In the rest frame, the wave parallel electric field still can be quite large. The remaining equations of motion are

$$\begin{aligned}&\frac{d\mathrm{v}_\perp}{dt}=-\frac{\rho}{2}\mathrm{v}_\perp\mathrm{v}_{||}+f+f^*\\&\frac{d\varphi}{dt}=-\Omega+\frac{i}{\mathrm{v}_\perp}f+c.c\end{aligned} \tag{10}$$

where

$$\begin{aligned}f=&-\frac{\mathrm{v}_{||}}{2}e^{i\varphi}\left[\mathrm{v}^{c-}_{d,s}+\rho\mathrm{v}^c_{dx}+\frac{\rho}{2}\mathrm{v}_\perp e^{i\varphi}\right.\\&+\left(\mathrm{v}^-_{d,s}+\frac{\partial\mathrm{v}^-_d}{\partial\mathrm{v}_{||}}\frac{d\mathrm{v}_{||}}{dt}+\rho\mathrm{v}_{dx}-\frac{B^{(1)-}}{ic}\right)e^{-i\vartheta}\\&\left.+\left(\mathrm{v}^{+*}_{d,s}+\frac{\partial\mathrm{v}^{+*}_d}{\partial\mathrm{v}_{||}}\frac{d\mathrm{v}_{||}}{dt}+\rho\mathrm{v}^*_{dx}-\frac{B^{(1)+*}}{ic}\right)e^{i\vartheta}\right].\end{aligned} \tag{11}$$

In expression (11), $B^{(1)}$ is the fraction of the wave magnetic field due to the ambient magnetic field inhomogeneity and the two last terms correspond to left- and right-hand polarized components of the electromagnetic wave. The term $d\mathrm{v}_{||}/dt$ is defined by formula (7). The particle dynamics description should also include the equation of the wave phase evolution,

$$\frac{d\vartheta}{dt}=\bar{\omega}. \tag{12}$$

Below the ambient magnetic field, the slowly changing electric field, the wave vector and amplitude variations are assumed to be weak on the scale l, of the particle Larmor radius, and particle oscillation in the wave field (see Ref. 8). In addition, l is considered to be small relative to the characteristic scale of inhomogeneity L introduced in connection with Eq. (8), i.e., l/L is considered to be a small parameter. Therefore, the main terms in Eqs. (7) and (10) are the terms proportional to the wave magnetic field, $B^\pm$, which are $kL\Omega/\omega$ times larger than other terms. These equations are averaged with second-order accuracy with respect to the small parameter l/L. With $T\sim L/\mathrm{v}$ this parameter also can be expressed as $1/\varpi T,\ 1/\Omega T$, where T is the characteristic time scale for a particle with velocity v to experience a spatial inhomogeneity. In the results presented below, the small terms by order of $(l/L)^2$ are omitted, while the large terms averaged with the accuracy to the second order are preserved.

As can be shown, in this approximation, the wave field does not affect the drift velocity that could be presented as

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$$\frac{d\vec{r}_\perp}{dt} = \vec{\mathrm{v}}_d^c + \mathrm{v}_\parallel \left[-\vec{e}_x \frac{\mathrm{v}_{dy,s}^c}{\Omega} + \vec{e}_y \left(\frac{\mathrm{v}_{dx}^c}{\Omega} \right)_s \right]. \tag{13}$$

Equations (10) and (11) for averaged perpendicular momentum can be written as

$$\frac{d\mathrm{v}_\perp}{dt} = \frac{\mathrm{v}_\perp \mathrm{v}_\parallel}{2} \left\{ -\rho + \left[\frac{\Omega |B^{(0)+}|^2}{(\Omega - \varpi)^3} \right]_s + \left[\frac{\Omega |B^{(0)-}|^2}{(\Omega + \varpi)^3} \right]_s \right\}. \tag{14}$$

Here the second and third terms correspond to left-hand and right-hand polarized waves, respectively. Rh terms can be obtained from the Lh terms by substitution $\omega \leftrightarrow -\omega$, $k \leftrightarrow -k$, $A^+ \leftrightarrow A^-$ and vice versa. Using such notations, the corresponding equation for the averaged parallel momentum change (ponderomotive force, $\mathrm{F}_\parallel$) is

$$\begin{aligned}
F_\parallel &= \mathrm{F}_\parallel^{(1)} + \mathrm{F}_\parallel^{(2)} \\
\mathrm{F}_\parallel^{(1)} &= \left[E_\parallel^c + \rho \left(\mathrm{v}_{dx}^{c2} + \frac{\mathrm{v}_\perp^2}{2} \right) \right] - \frac{1}{2\omega} \left[\frac{\Omega |E^-|^2}{(\Omega + \varpi)^2} \right]_s - \frac{1}{2\omega^2} \left[\frac{\varpi^2 |E^-|^2}{(\Omega + \varpi)^2} \right]_s + (Rh \to Lh) \\
\mathrm{F}_\parallel^{(2)} &= \left[E_\parallel^c + \rho \left(\mathrm{v}_{dx}^{c2} + \frac{\mathrm{v}_\perp^2}{2} \right) \right] \left[-\frac{\Omega |B^{(0)-}|^2}{(\Omega + \varpi)^3} + (Rh \to Lh) \right] - \frac{\mathrm{v}_\perp^2}{4} \frac{\left(\Omega |B^{(0)-}|^2 \right)_s}{\Omega (\Omega + \varpi)^2} + (Rh \to Lh) + \frac{\rho \mathrm{v}_\perp^2}{2} \frac{B^{+*}B^- + B^+ B^{-*}}{\Omega^2 - \varpi^2}.
\end{aligned} \tag{15}$$

The system of equations (13)–(15) describes the averaged particle motion. Equation (15) indicates that the affect of the left-hand and right-hand polarized waves is not additive and includes their joint action on the particle, described by the last term in $\mathrm{F}_\parallel^{(2)}$. Note that this term can be readily rewritten as a sum of products of terms related to the left- and right-hand polarized waves. The same is true for similar terms in the equation for energy change below. So, the results are applicable for the calculation of ponderomotive effects if the waves with different polarizations also have different frequencies and wave vectors. The equation for the perpendicular velocity leads to the conservation of the modified particle magnetic moment,

$$\frac{\mathrm{v}_\perp^2}{B_0} \left[1 - \frac{\Omega |B^{(0)+}|^2}{(\Omega - \varpi)^3} - \frac{\Omega |B^{(0)-}|^2}{(\Omega + \varpi)^3} \right] = const. \tag{16}$$

This equation can be supplemented by the equation for the evolution of particle energy, $\varepsilon = mu^2/2$, $d\varepsilon/dt \equiv P$.

$$\begin{aligned}
P &= P^{(1)} + P^{(2)} \\
P^{(1)} &= \mathrm{v}_\parallel \left(E_\parallel^c + \rho \mathrm{v}_{dx}^{c2} + \frac{\mathrm{v}_{d,s}^{c2}}{2} \right) - \frac{\mathrm{v}_\parallel}{2\omega} \left[\frac{\Omega |E^-|^2}{(\Omega + \varpi)^2} \right]_s + (Rh \to Lh) \\
P^{(2)} &= \left[E_\parallel^c + \rho \left(\mathrm{v}_{dx}^{c2} - \frac{\mathrm{v}_\perp^2}{2} \right) \right] \left[-\frac{k\Omega |E^-|^2}{\omega (\Omega + \varpi)^3} + (Rh \to Lh) \right] + \left(\rho \mathrm{v}_{dx}^{c2} + \frac{\mathrm{v}_{d,s}^{c2}}{2} \right) \frac{\mathrm{v}_\parallel}{2} \\
&\times \left\{ \frac{|B^{(0)-}|^2}{(\Omega + \varpi)^2} - \frac{1}{2} B^{(0)+} B^{(0)-*} \mathrm{v}_d^{c-} \left(\mathrm{v}_{d,s}^{c-} + \rho \mathrm{v}_{dx}^c \right) \left[\frac{1}{(\Omega + \varpi)^2} + \frac{1}{(\Omega - \varpi)^2} \right] + (Rh \to Lh) \right\}.
\end{aligned} \tag{17}$$

Here P represents the ponderomotive power of the wave. The terms in the last line of Eq. (17) are in the order of the omitted terms, if the inequality $\frac{\mathrm{v}_\parallel}{\mathrm{v}_{ph}} kL \gg 1$ is violated. Here v_{ph} is the wave phase velocity, ω/k.

III. DISCUSSION

In Sec. II, we presented expressions for the averaged parallel (ponderomotive) force (Eq. (15)) and energy change (Eq. (17)) as the sum of two terms. The terms $F_\parallel^{(1)}$ and $P^{(1)}$ in these expressions represent the previously known results. For example, in the absence of the electromagnetic wave the equations coincide with the corresponding drift equations in the presence of a strong electric field presented by Northrop.[10] The terms $F_\parallel^{(2)}$ and $P^{(2)}$ in the corresponding relations are the new additions obtained in this paper.

The term $F_\parallel^{(1)}$, in Eq. (15), and $P^{(1)}$, in Eq. (17), also include the known results for the ponderomotive forces and their energy change. They can be compared to the results,[8,11,12] where the drift equations were obtained for a particle in a weak circularly polarized wave ($cE_\perp/B_0 \ll \mathrm{v}$, inequality opposite to Eq. (3)) in the absence of slowly changing electric fields ($E_\parallel^c = 0, \mathrm{v}_d^c = 0$). It was found that the modified magnetic moment $(\mathrm{v}_\perp^2 - \mathrm{v}_d^{t2})/B_0$ is conserved. In the present work, however, the drift energy $\mathrm{v}_d^{t2}/2$ is excluded by the definition of the particle perpendicular velocity (Eq. (5)). So, both expressions for the magnetic moment coincide, if the wave magnetic field in Eq. (16) is neglected. In both cases, the mirror force ($\sim \rho \mathrm{v}_\perp^2/2$) is dependent on the corresponding adiabatic invariants, as can be found by combining the terms $\rho \mathrm{v}_\perp^2/2$ in Eq. (15) and expression (16). The second and third terms in the parallel force

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$F_{\parallel}^{(1)}$ (Eq. (15)) and energy exchange, $P^{(1)}$ (Eq. (17)) are the same as in paper by Khazanov *et al.*[8] and in Refs. 11 and 12, if the terms kv_s in $F_{\parallel}^{(1)}$ and $P^{(1)}$ are omitted.

The new terms in expression (15), for $F_{\parallel}$ (force $F_{\parallel}^{(2)}$), and (17), for P (power $P^{(2)}$), present the ponderomotive correction due to the wave magnetic field intensity. The correction to the energy rate $P^{(2)}$, however, also contains terms that are proportional to the wave Poynting vector that will be discussed below.

There is another limit that was considered in a past derivation of ponderomotive forces, which belongs to the case of homogeneous plasma, as discussed by Khazanov *et al.*[4] They considered the motion of a particle accelerated by gravity along a homogeneous magnetic field in the presence of a circularly polarized wave. The terms that remain in the limit of a homogeneous non-drifting plasma in the equations for parallel momentum and energy change (first term in $F_{\parallel}^{(2)}$, Eq. (15), and first term in $P^{(2)}$, Eq. (17)), are exactly the same as obtained by Khazanov *et al.*[4] if the gravitational force is replaced by the electric force, $E_{\parallel}^{c}$. They found that the particle perpendicular velocity, induced by the wave electric field, is orthogonal to this field if the parallel velocity is constant and the energy exchange between the wave and the particle is inhibited. Particle acceleration along the ambient magnetic field destroys this orthogonality. As can be seen from the terms above (Eqs. (15) and (17)), the nature of the acceleration is not important since the magnetic field curvature also changes the parallel velocity and leads to a similar effect. Thus, the new additional ponderomotive force that we found in the present work can be presented as

$$F_{\parallel}^{new1} = \left[E_{\parallel}^{c} + \rho\left(v_{dx}^{c2} + \frac{v_{\perp}^{2}}{2}\right)\right]\left[-\frac{\Omega|B^{(0)-}|^{2}}{(\Omega+\varpi)^{3}} + (Rh \rightarrow Lh)\right]. \tag{18}$$

The term with $E_{\parallel}^{c}$ is similar to the term with gravitational acceleration in Khazanov *et al.*,[4] and, therefore, is valid even if the restriction on $E_{\parallel}^{c}$ that follows from the condition $S_0 \sim \rho$ is violated.

The corresponding energy change due to this new ponderomotive force Eq. (18) can be extracted from the expression of $P^{(2)}$ (Eq. (17)) and presented as

$$P^{new(1)} = \left[E_{\parallel}^{c} + \rho\left(v_{dx}^{c2} - \frac{v_{\perp}^{2}}{2}\right)\right]\left[-\frac{k\Omega|E^{-}|^{2}}{\omega(\Omega+\varpi)^{3}} + (Rh \rightarrow Lh)\right]. \tag{19}$$

It should be noted that the averaged equations do not depend on the parallel wave electric field since their contribution only enters as higher order corrections.

The additional new terms in the parallel momentum (Eq. (15)) and energy exchange (Eq. (17)) are the terms with the wave magnetic field intensity,

$$F_{\parallel}^{new2} = -\frac{v_{\perp}^{2}}{4}\frac{\left(\Omega|B^{(0)-}|^{2}\right)_{s}}{\Omega(\Omega+\varpi)^{2}} + (Rh \rightarrow Lh) - \frac{\rho v_{\perp}^{2}}{2}\frac{B^{+*}B^{-} + B^{+}B^{-*}}{\varpi^{2}-\Omega^{2}} \tag{20}$$

and

$$P^{new2} = \left(\rho v_{dx}^{c2} + \frac{v_{d,s}^{c2}}{2}\right)\frac{v_{\parallel}}{2} \times \left\{\frac{|B^{(0)-}|^{2}}{(\Omega+\varpi)^{2}} - \frac{1}{2}B^{(0)+}B^{(0)-*}v_{d}^{c-}\left(v_{d,s}^{c-} + \rho v_{dx}^{c}\right) \times \left[\frac{1}{(\Omega+\varpi)^{2}} + \frac{1}{(\Omega-\varpi)^{2}}\right] + (Rh \rightarrow Lh)\right\}. \tag{21}$$

These new terms entered in the parallel force $F_{\parallel}^{(2)}$ (Eq. (15)) and power $P^{(2)}$, Eq. (17), are of the order of $v_{\perp}^{2}/v_{ph}^{2}$, and v_{d}^{c2}/v_{ph}^{2}, respectively, compared to the terms dependent on the wave electric field. According to Streltsov and Lotko,[13] condition $v_{\perp}^{2}/v_{ph}^{2} > 1$ is met for electrons and Alfvén waves at the Earth's auroral field lines starting from a distance of about 5 Earth radii. The ion and electron thermal velocity can be comparable to the phase velocity of Alfvén waves propagating in a hot plasma. So, the condition $v_{\perp}^{2}/v_{ph}^{2} > 1$ is also satisfied in the regions of quiet Alfvén wave activity that occur in the main bodies of solar wind streams or on their trailing edges. Here (Belcher and Davis[14]), the plasma beta is $\beta = 8\pi nT/B_0^2 \sim 0.4$ and $B/B_0 \sim 0.2$. The ratio of the new term in Eq. (19) to the wave term in the power $P^{(1)}$ (Eq. (17)) is kv/Ω. So, the new terms in Eqs. (15) and (17) are more important for the fast particles. The terms with the parallel component of the electric field $E_{\parallel}^{c}$ in the energy equation (17) are of the order of $v_{\parallel}$ and $|E|^2/(v_{ph}\Omega^2)$, respectively, and for particles with small parallel velocity the energization due to the wave can be larger than that due to the parallel electric field $E_{\parallel}^{c}$.

It should be stressed that the new terms can be essential not only for hot plasma and fast particles. The estimated ratio of the new term in Eq. (19) with $E_{\parallel}^{c}$ to the wave term in the energy rate $P^{(1)}$ (Eq. (17)) is kv/Ω. This estimation is based on the condition $E_{\parallel}^{c}L \sim v^2/2$ (or $S_0 \sim \rho$ (Eq. (8)). In the paper by Khazanov *et al.*,[4] the similar ponderomotive term with gravitational acceleration instead of $E_{\parallel}^{c}$ is found without an analogous restriction on gravitation. The necessary restriction on the parallel electric field is $E_{\parallel}^{c} \ll E_{\perp}^{c}$, required by the applicability of the averaging procedure of Morozov and Solov'ev.[9] The two terms under consideration are proportional to $c\frac{E_{\perp}^{c}}{B_0}\frac{E_{\parallel}^{c}}{E_{\perp}^{c}}$ and $\frac{v_{\parallel}}{kL}$, respectively, and their ratio depends on a specific problem that is beyond consideration in this paper.

As it is known, ponderomotive effects strongly depend on the spatio-temporal variation of the wave intensity. If the waves are a sequence of wave packets, an additional time dependent ponderomotive force, proportional to the wave intensity time derivative, arises (Khazanov *et al.*,[8] and Shukla *et al.*[15]). The energy gain due to this force is comparable to the term defined by the wave intensity space variation (second term in $P^{(1)}$ of Eq. (17)) and, therefore, all these terms can be comparable.

IV. CONCLUSION

To summarize, we emphasize several points of this study. We considered the ponderomotive effects of an electromagnetic wave for a particle drifting in a plane ambient

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magnetic field in the presence of a parallel electric field under the conditions $c\frac{E_\perp^c}{B_0} \sim \mathrm{v}$, $c\frac{E_\perp}{B_0} \sim \mathrm{v}$. The wave propagates along the background magnetic field line. The fields, the wave amplitude, and the wave vector are slowly changing functions of the distance along the magnetic field line. The modified first adiabatic invariant is found to be determined by (Eq. (16)). Additional terms in the ponderomotive force, Eqs. (18) and (20), as well as in the energy rate, Eqs. (19) and (21), all dependent on the wave magnetic field, were calculated. These terms can be essential for hot plasmas or fast particles with the velocities comparable to the wave phase velocity. Particle acceleration along the magnetic field line due to the parallel electric field, the magnetic line curvature or the changing drift velocity results in additional ponderomotive effects independent of the spatio-temporal derivatives of the wave intensity. The input of the new term in the particle energy rate, defined by the particle acceleration due to the parallel electric field, can be of the same order as the ponderomotive terms defined by the space and time derivatives of the wave intensity. The results presented here can be useful for studies of space plasma phenomena.

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