Statistical modeling and analysis of multi-stranded coronal loops (part 2)

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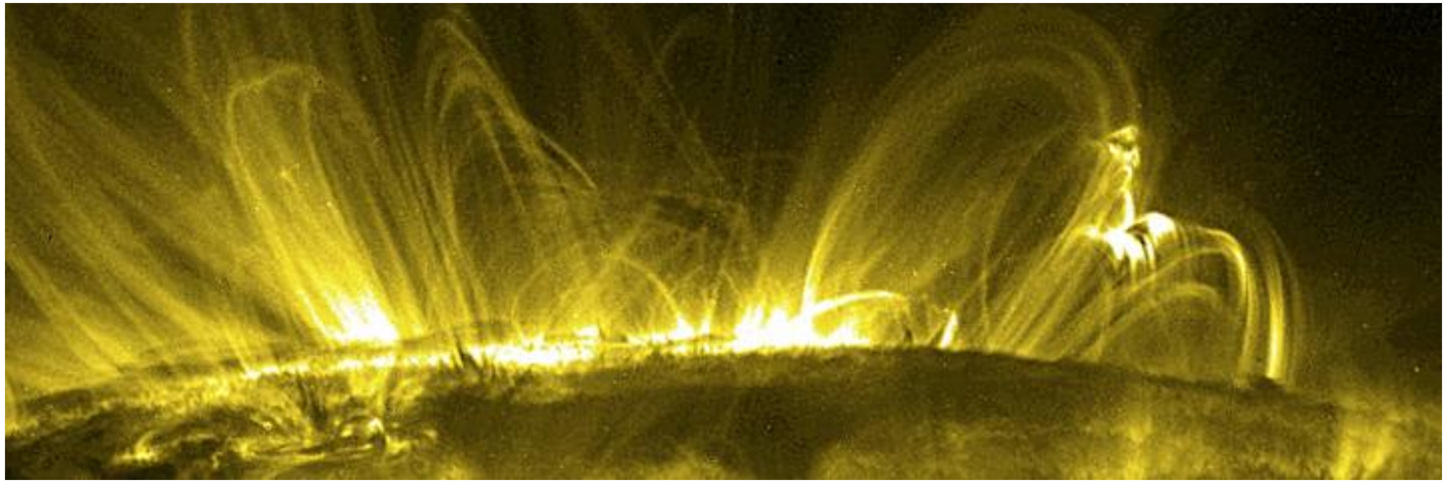
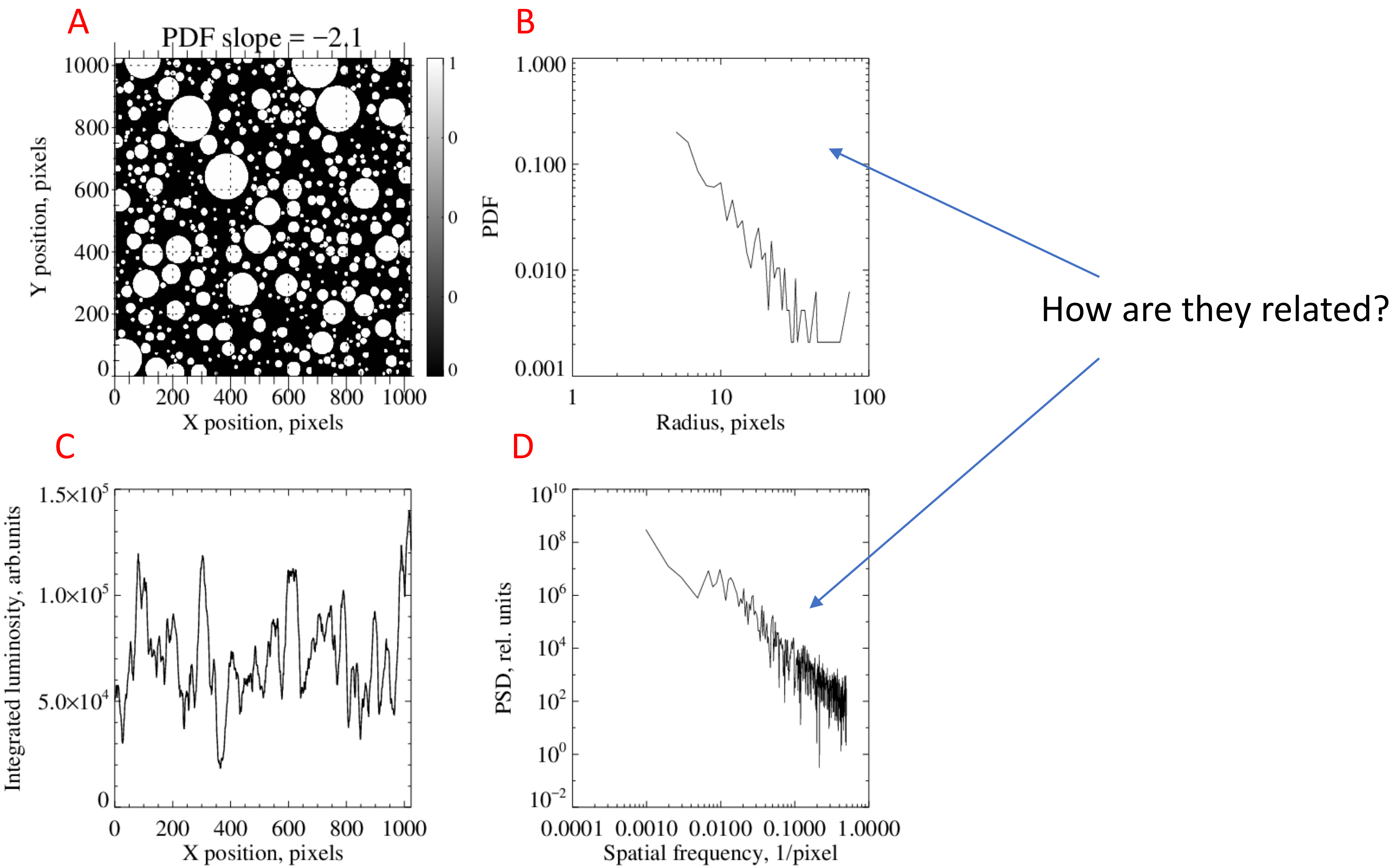


Modeling emission signal from unresolved overlapping loops



(Based on: Jensen at al., *Phys Rev B* 1989; Kertesz & Kiss, *J Phys A* 1990; Manna et al., *J Stat Phys* 1990)

Net emission impact from an individual strand of size D_s:

$$s = \int_x^{x+D_s} I_s(x_\perp) dx_\perp \qquad (1)$$

Fourier transform of $I_s(x_\perp)$:

$$\tilde{I}_s(k_\perp) = \int \exp(ik_\perp x_\perp) I_s(x_\perp) dx_\perp \qquad (2)$$

Mean energy density spectrum of strands of characteristic transverse linear size (diameter) D_s:

$$\langle E_s(k_\perp) \rangle = \left\langle |\tilde{I}_s(k_\perp)|^2 \right\rangle \qquad (3)$$

Lorentzian energy density spectrum
(asymptotic solution for small $k_\perp$, normalization condition $\tilde{I}_s(k_\perp = 0) = s$):

$$\langle E_s(k_\perp) \rangle = \frac{s^2}{1 + k_\perp^2 D_s^2} \qquad (4)$$

(W. Schottky, Phys. Rev. 28 (1926) 74)

Total power density spectrum is the weighted sum of individual contributions:

$$\langle S(k_\perp) \rangle \sim \int p(s) \frac{s^2 ds}{1 + k_\perp^2 D_s^2} \qquad (5)$$

in which $p(s)$ is the density distribution of strands of net emission impact s.

Scaling assumptions:

$$D_s \sim s^{\gamma} \tag{6}$$

$$p(s) \sim s^{-\tau} \tag{7}$$

$$p(D_s) \sim D_s^{-\alpha} \tag{8}$$

(density distribution of strands of size D_s).
Power spectrum after substitutions:

$$S(k_\perp) \sim \int_{s_1}^{s_2} \frac{s^{2-\tau}}{1 + k_\perp^2 s^{2\gamma}} ds \sim \frac{1}{k_\perp^{(3-\tau)/\gamma}} \int_{k_\perp D_{s1}}^{k_\perp D_{s2}} \frac{u^{(3-\gamma-\tau)/\gamma}}{1+u^2} du \tag{9}$$

where s_1 (s_2) and D_{s1} (D_{s2}) are net emission impacts and sizes at the lower (upper) spectral cut-off. The integral is convergent as $D_{s2} \to \infty$ under the condition

$$2\gamma + \tau > 3 \tag{10}$$

which yields

$$S(k_\perp) \sim \frac{1}{k_\perp^{(3-\tau)/\gamma}} \sim \frac{1}{k_\perp^{\beta}} \tag{11}$$

where

$$\beta = \frac{3-\tau}{\gamma} \tag{12}$$

Otherwise, if

$$2\gamma + \tau \leq 3, \tag{13}$$

the upper cut-off contributes to the dependence since the weight of the $D_s >> 1/k_\perp$ modes becomes important:

$$S(k_\perp) \sim (k_\perp D_{s2})^{(3-2\gamma-\tau)/\gamma} \frac{1}{k_\perp^{(3-\tau)/\gamma}} = D_{s2}^{(3-2\gamma-\tau)/\gamma} \frac{1}{k_\perp^2} \tag{14}$$

and so the low-frequency behavior is always described by

$$\beta = 2 \tag{15}$$

(with logarithmic correction for $2\gamma + \tau = 3$).

From probability conservation $p(D_s)dD_s \sim p(s)ds$ one gets

$$\tau = 1 - \gamma(1 - \alpha) \tag{16}$$

allowing one to rewrite the convergence conditions in terms of the size distribution exponent α:

$$\alpha > \frac{2}{\gamma} - 1 \tag{17}$$

If the above condition is met, the spectral exponent follows the scaling law

$$\beta = \frac{2}{\gamma} + 1 - \alpha \tag{18}$$

and equals 2 otherwise as shown before.

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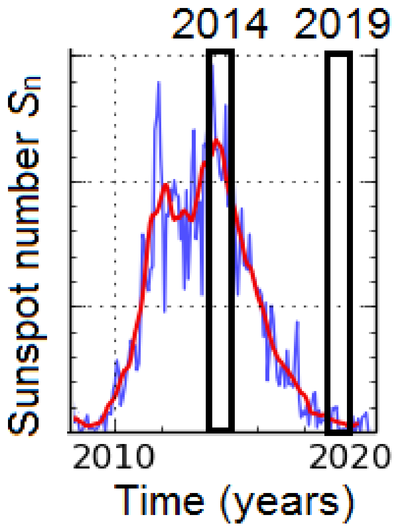
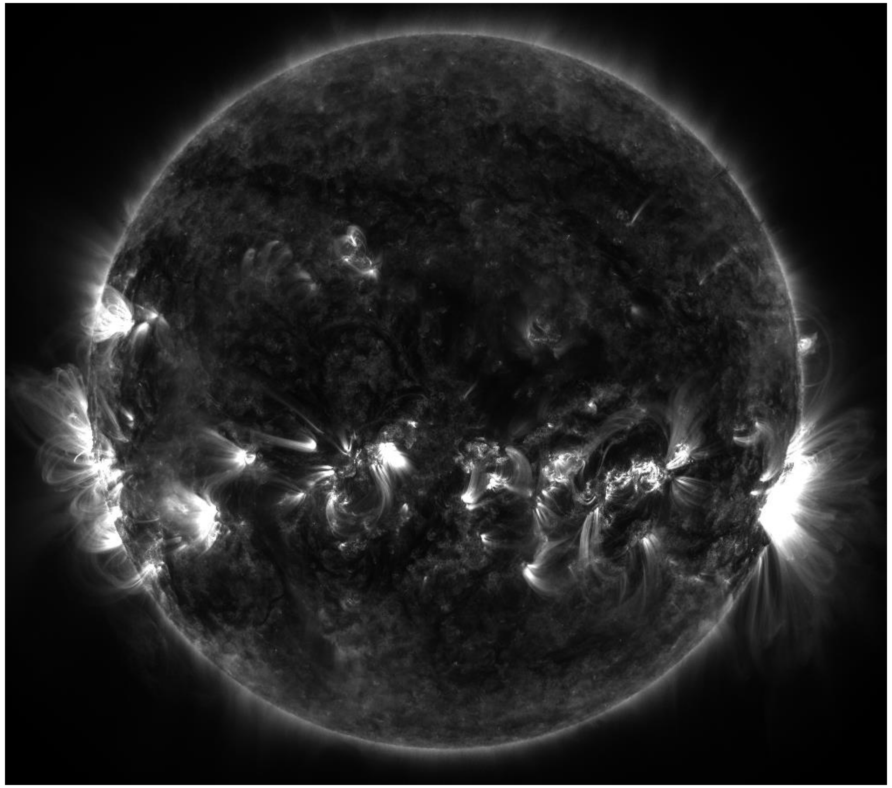
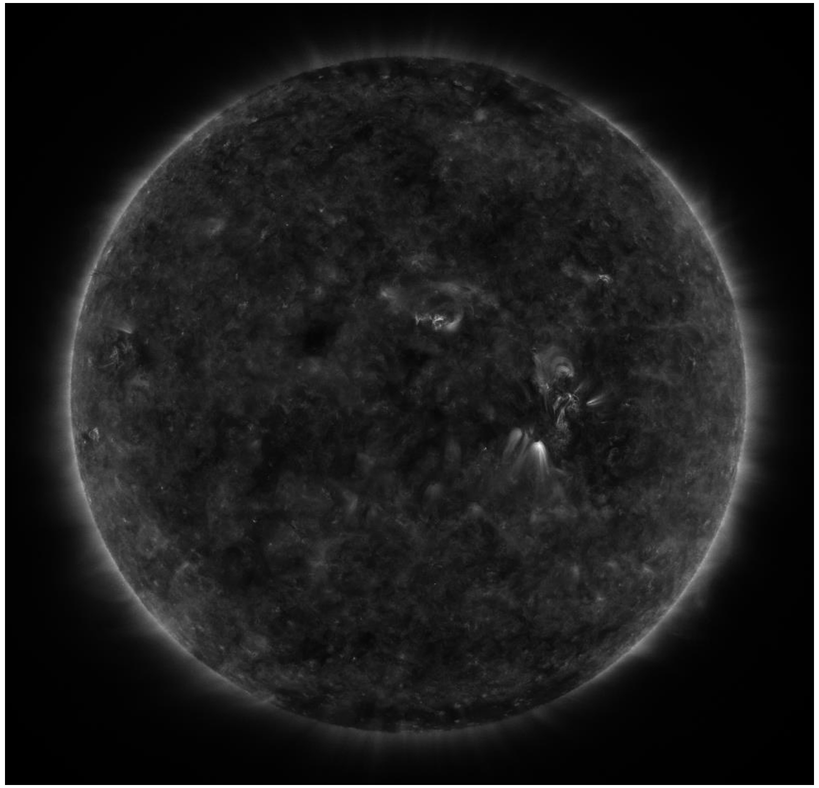


2014 – 01 – 01 00:00:11.34



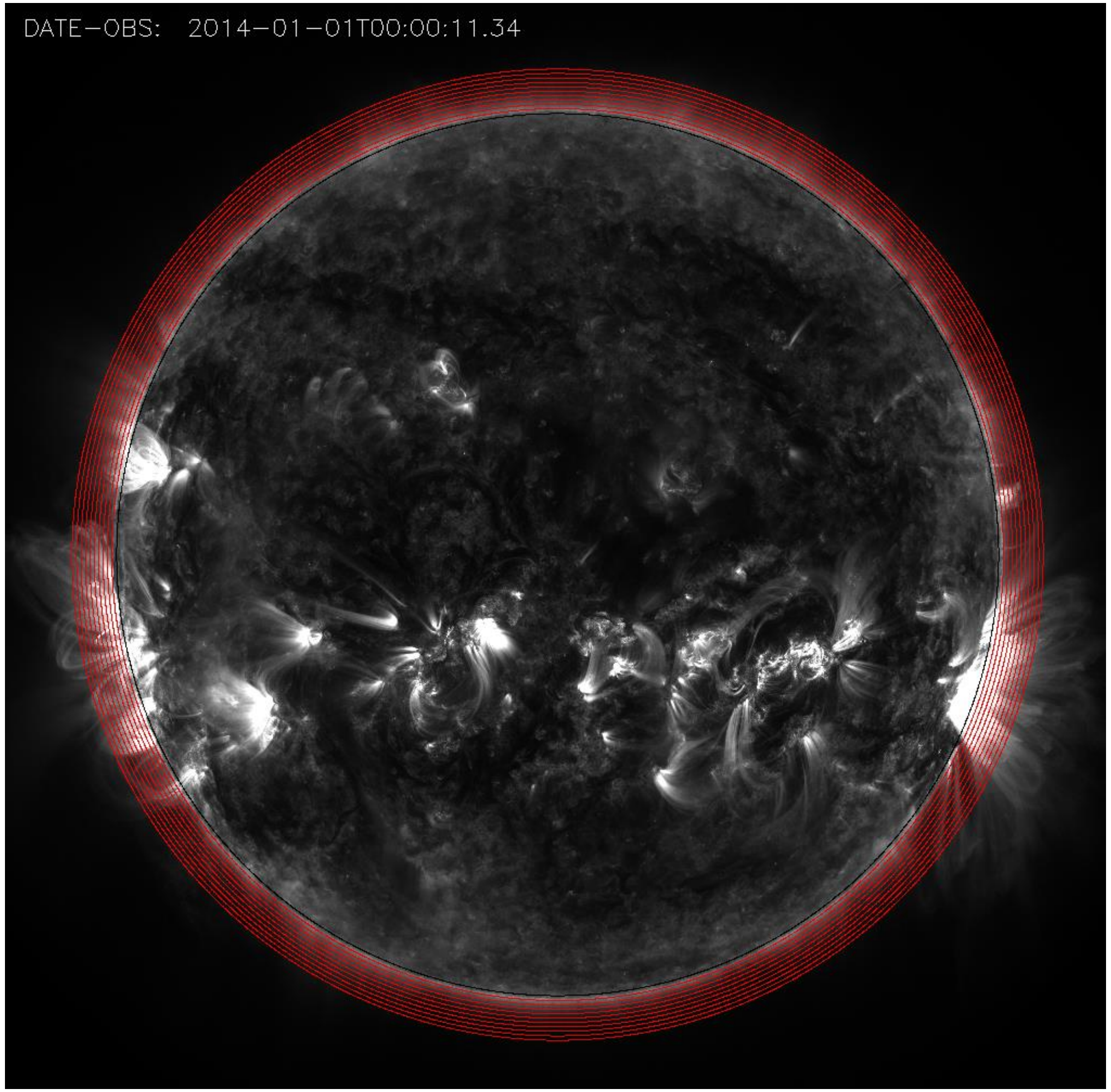
2014: n = 120

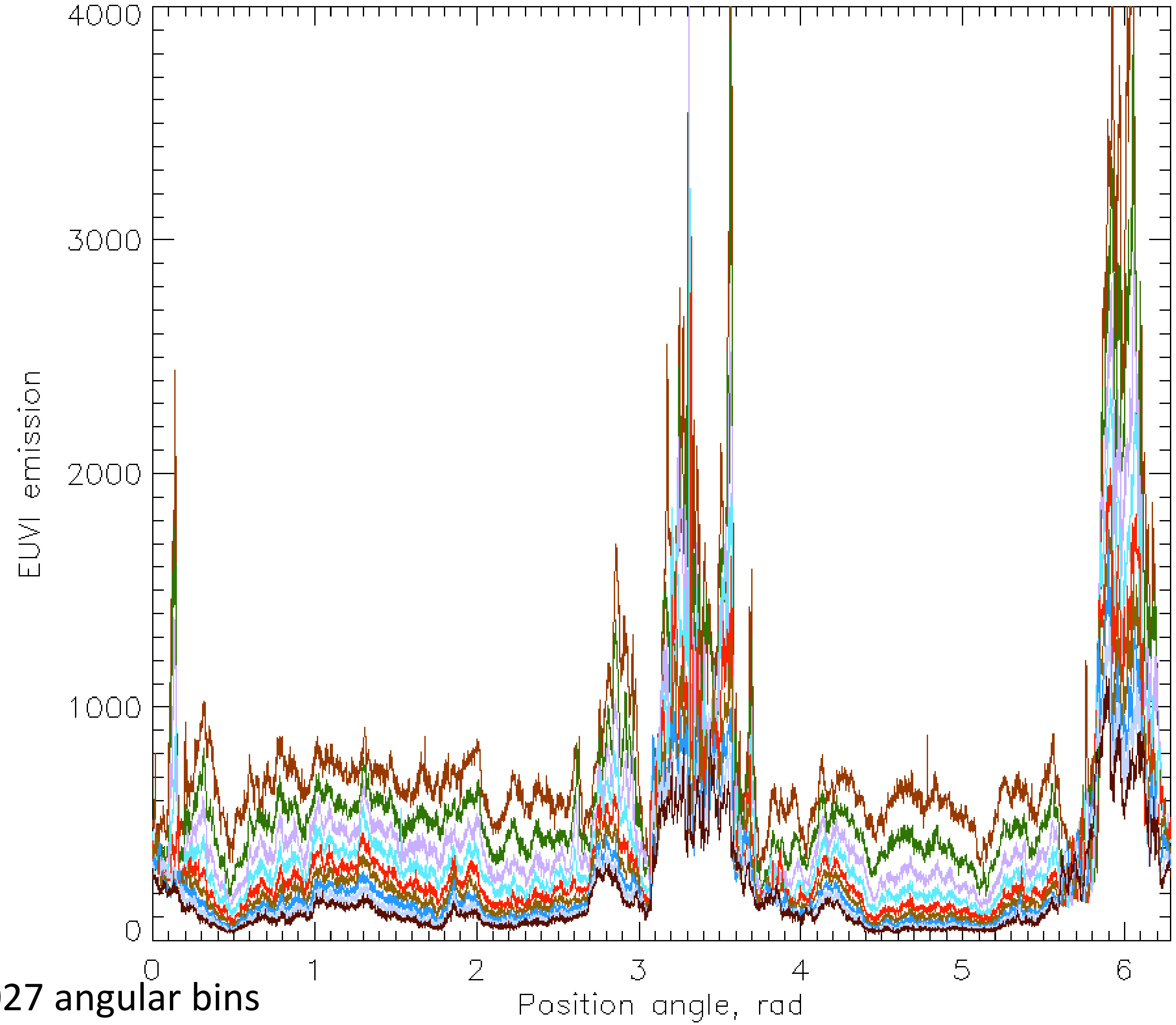
2019 – 01 – 01 00:00:09.35



2019: n = 120

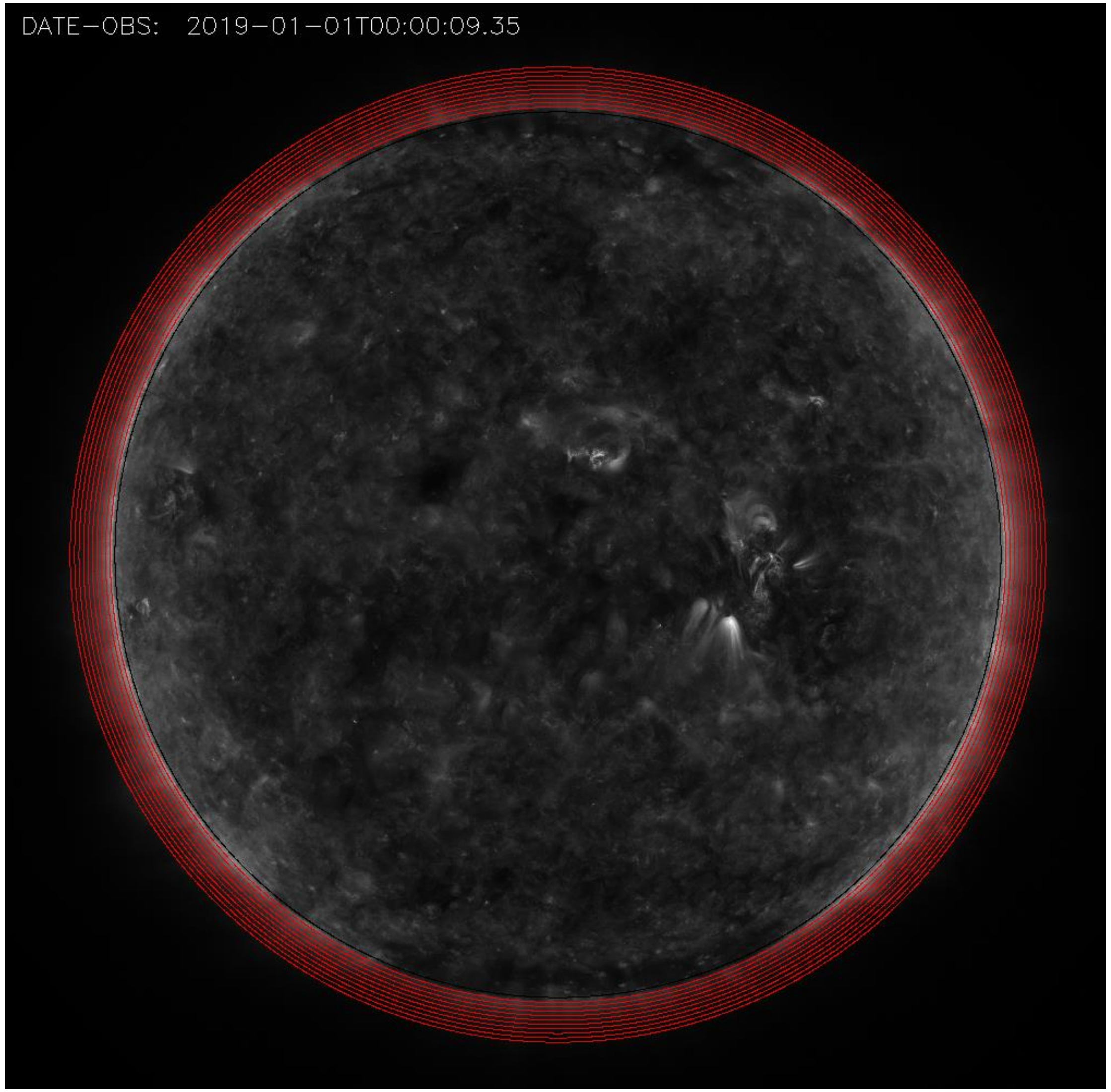
Solar maximum example: 2014/01/01

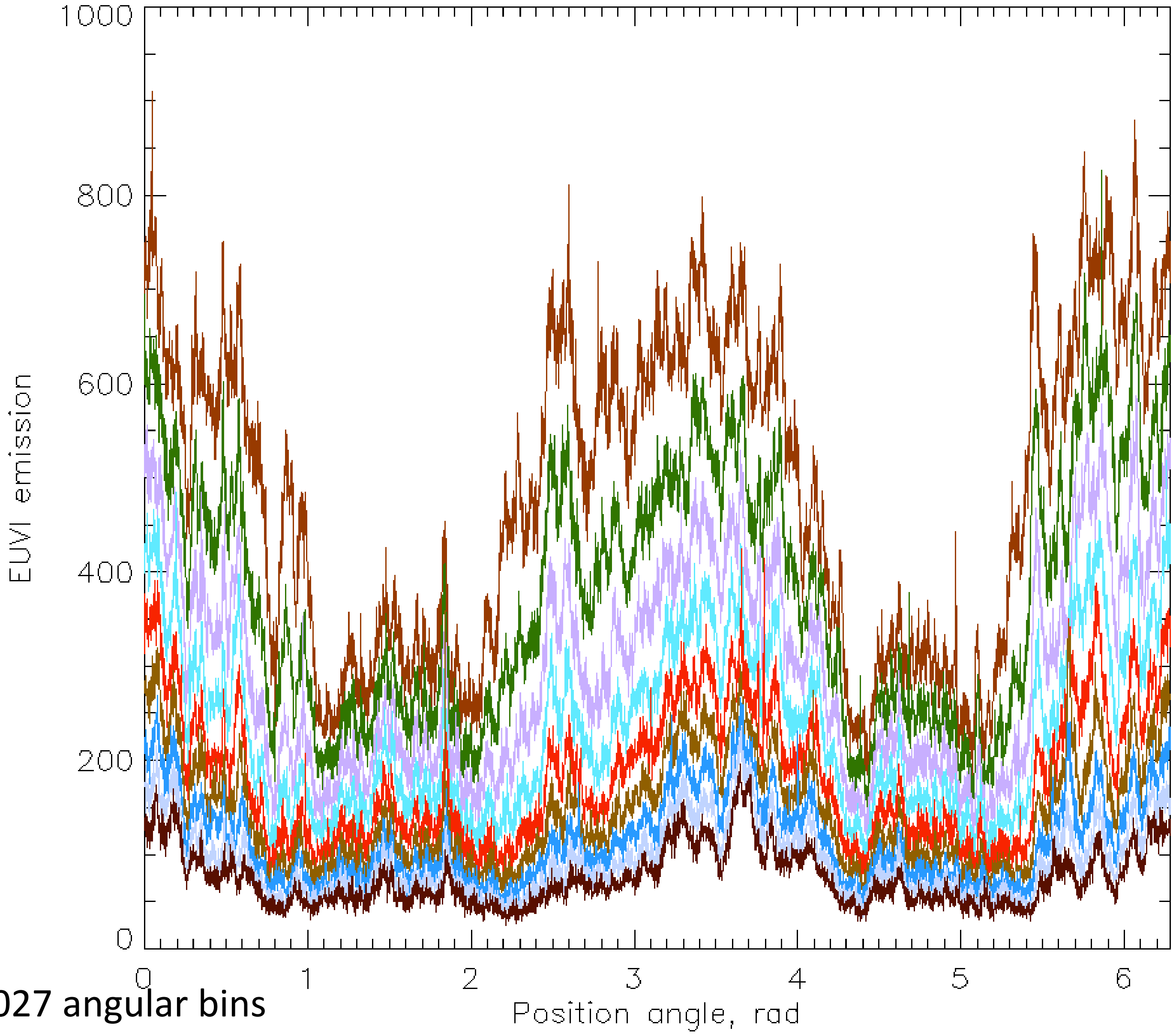




R_{min} = 1.01 R_S, R_{max} = 1.11 R_S, 100 azimuthal profiles, 5027 angular bins

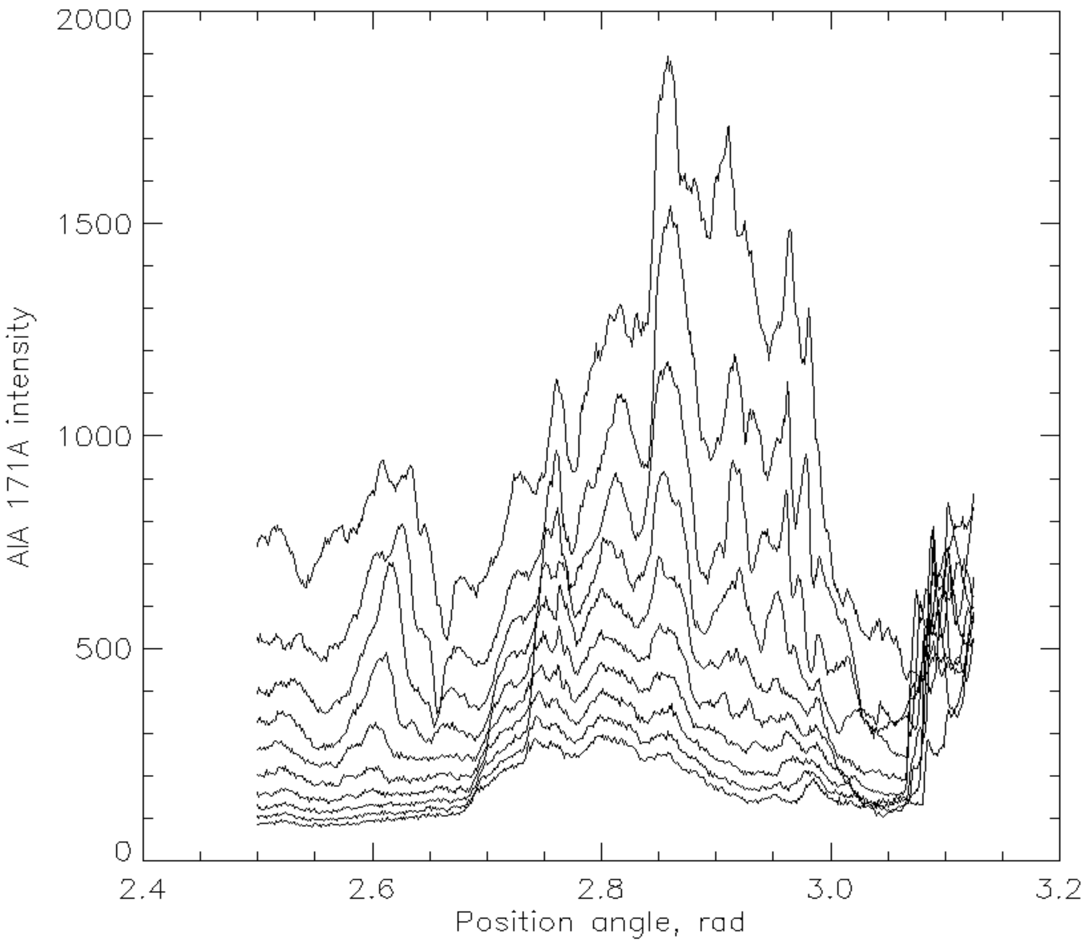
Solar minimum example: 2019/01/01



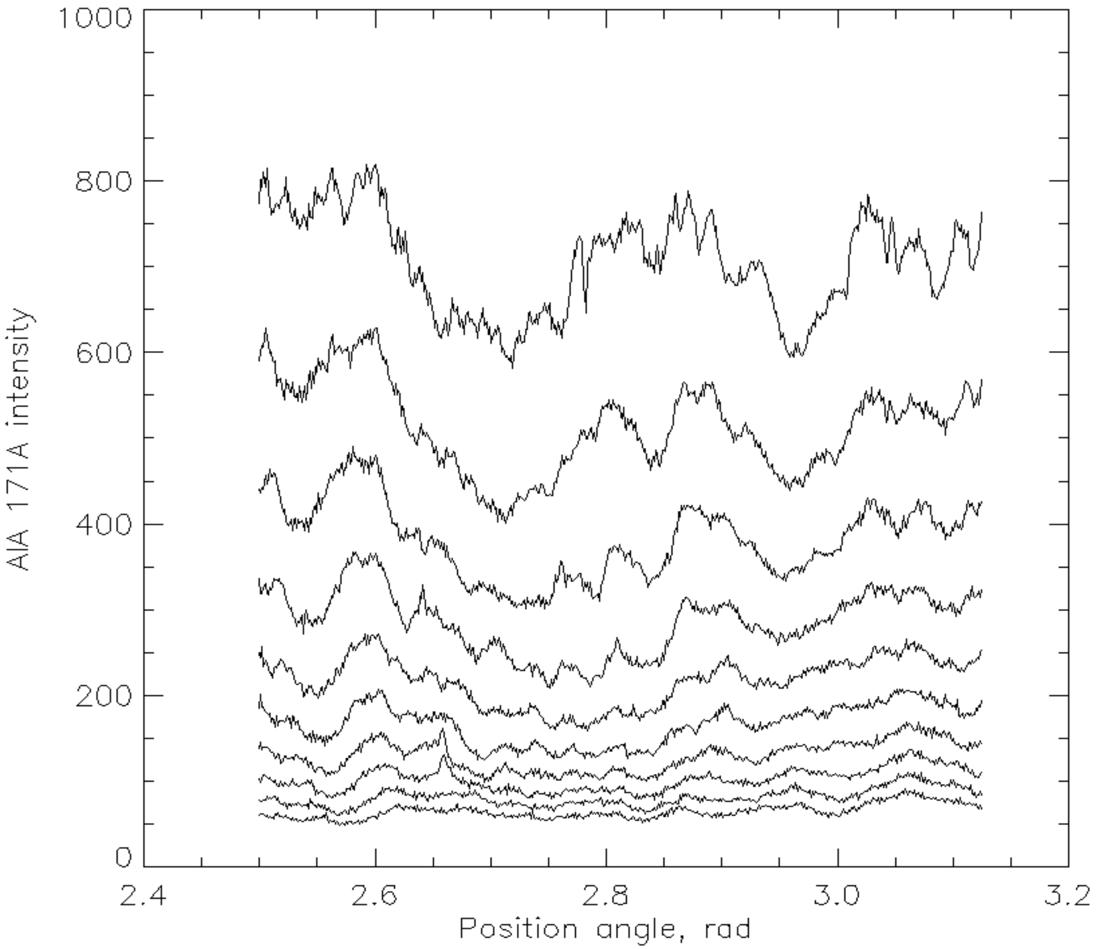


$R_{min} = 1.01\ R_S$, $R_{max} = 1.11\ R_S$, 100 azimuthal profiles, 5027 angular bins

R = 1.02 – 1.03 R_s



2000
1500
1000
500
0
2.4
2.6
2.8
3.0
3.2
AIA 171A intensity
Position angle, rad



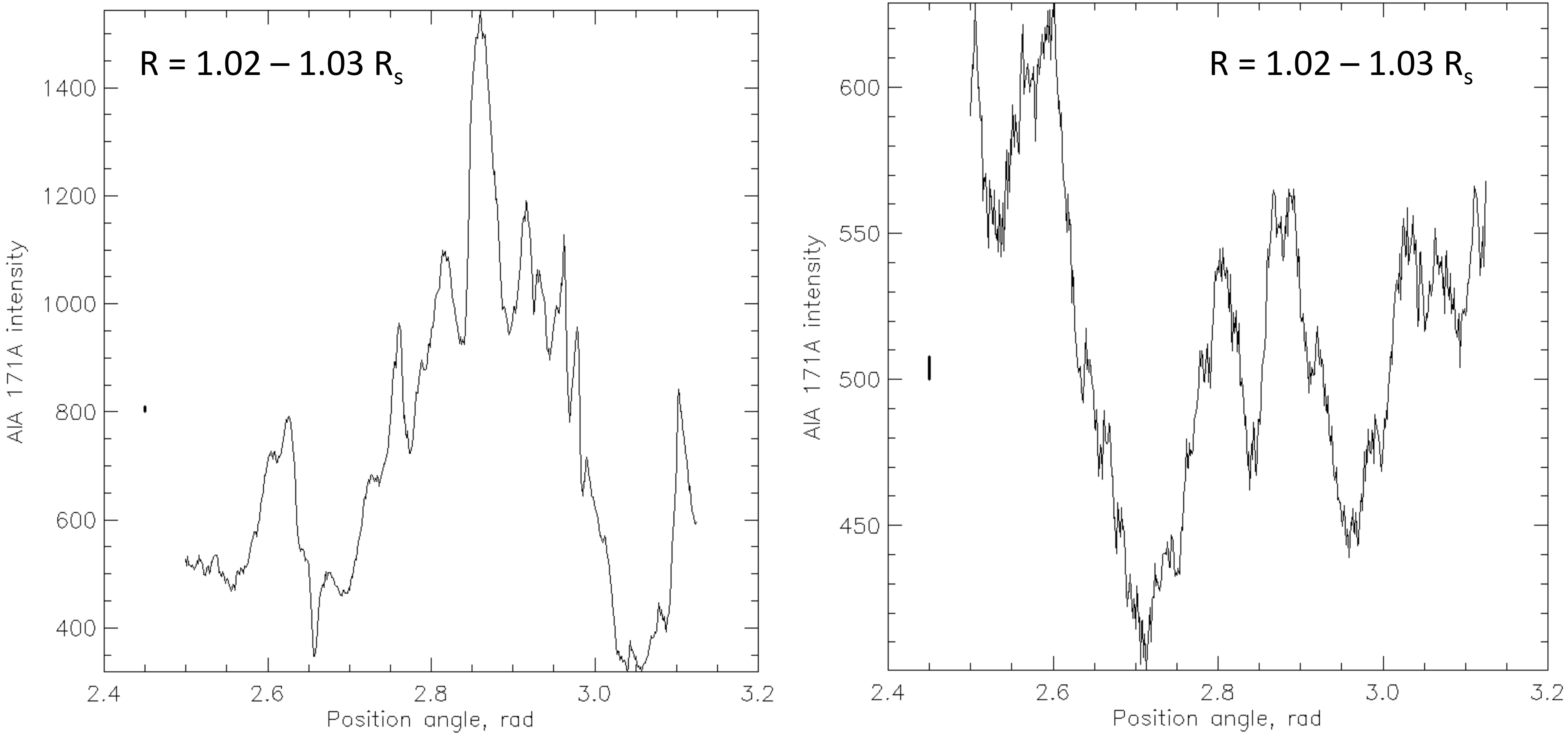
1000
800
600
400
200
0
2.4
2.6
2.8
3.0
3.2
AIA 171A intensity
Position angle, rad

Q = **AIA_BP_ESTIMATE_ERROR**(mean(res_2014.cut_arr[0,1].intensity[2000:2500]), 171, /loud)

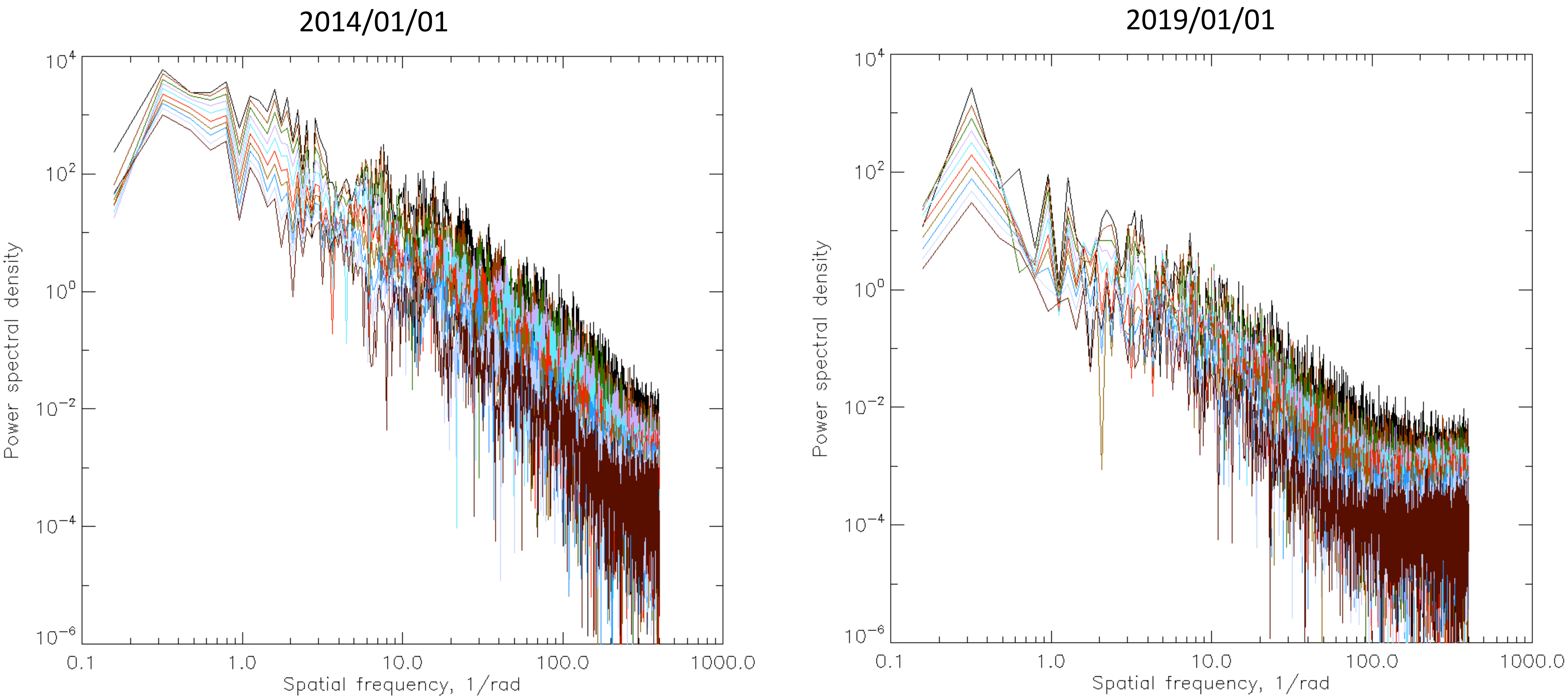
Counts [DN]	RMS Error	SNR	Shot	Dark	Read	Quant	Compress	Chianti	Calibrat
724.87	28.61	25.33	28.52	0.18	1.15	0.29	1.98	0.00	0.00

q = **AIA_BP_ESTIMATE_ERROR**(mean(res_2019.cut_arr[0,1].intensity[2000:2500]), 171, /loud)

Counts [DN]	RMS Error	SNR	Shot	Dark	Read	Quant	Compress	Chianti	Calibrat
514.13	24.11	21.33	24.02	0.18	1.15	0.29	1.67	0.00	0.00

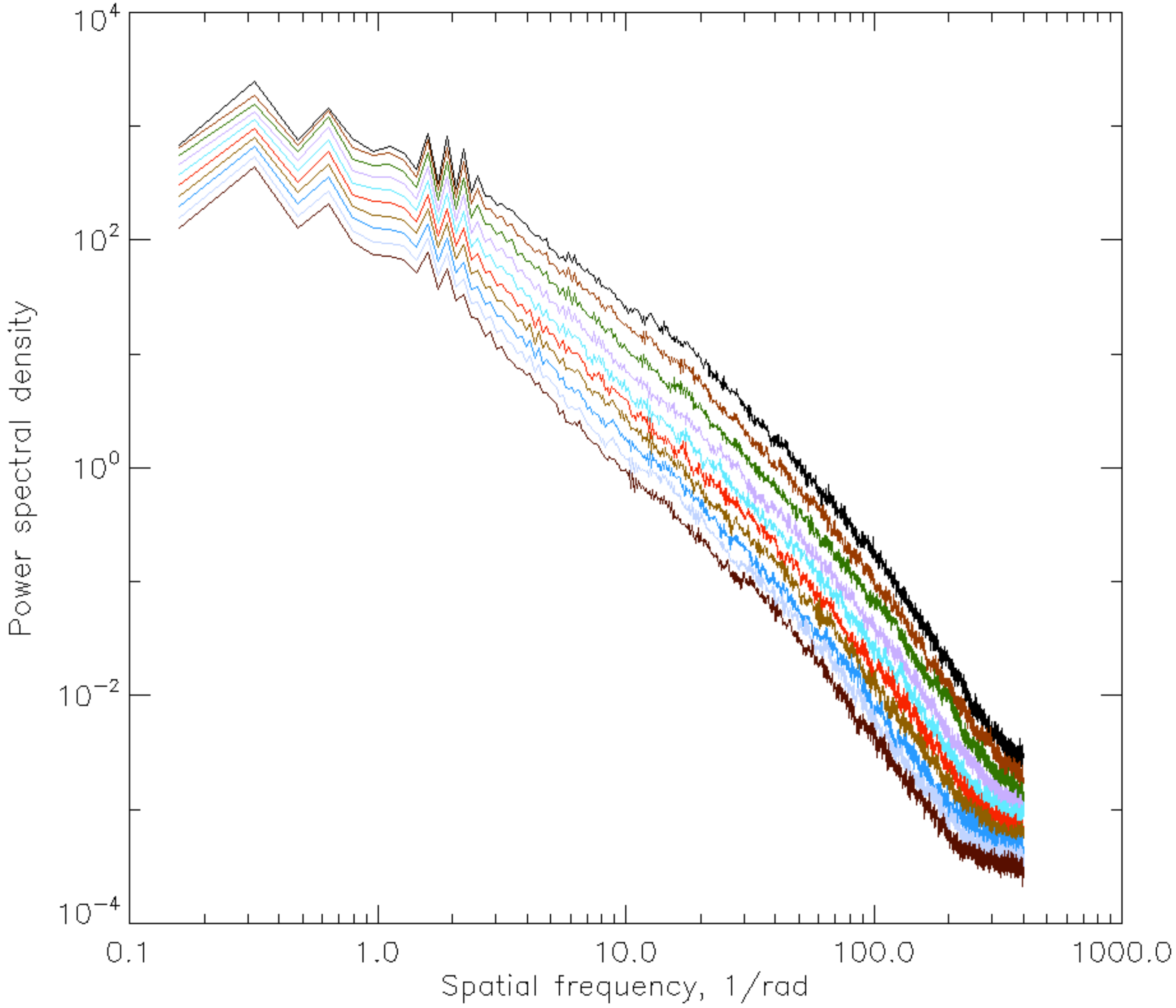


Single day

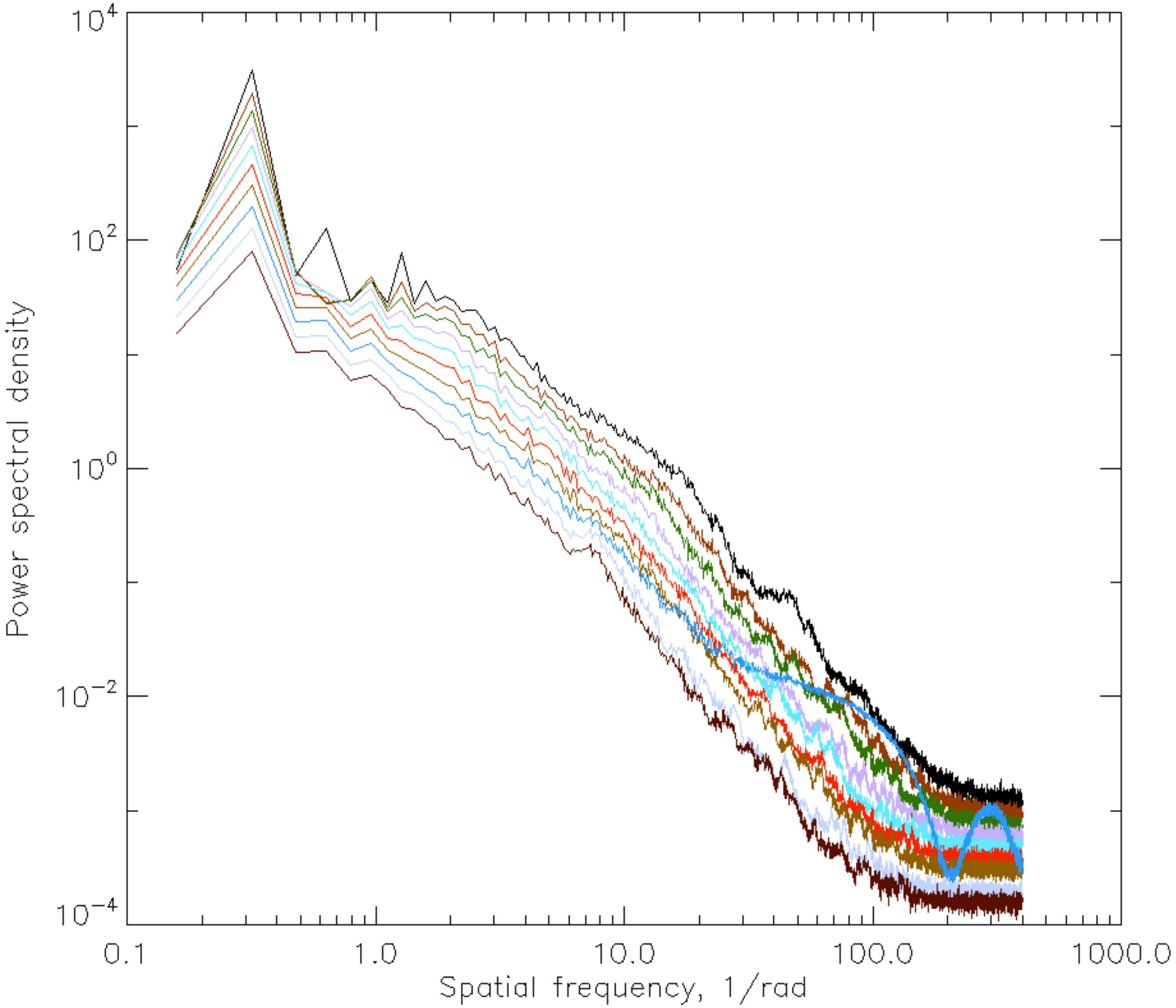

2014/01/01
2019/01/01
Power spectral density
Spatial frequency, 1/rad
0.1
1.0
10.0
100.0
1000.0
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}

Yearly averages

2014, averaged over full profiles

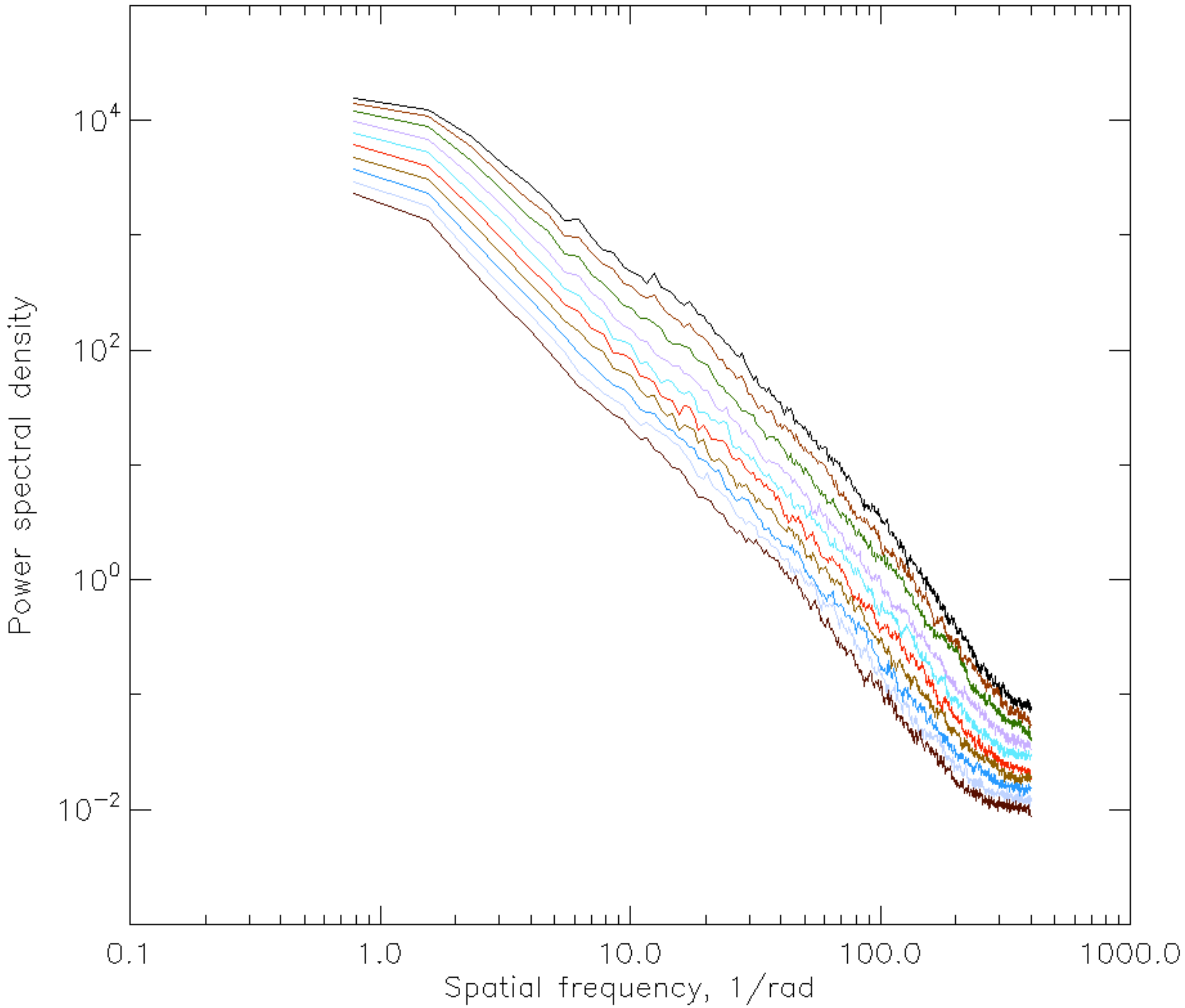


2019, averaged over full profiles

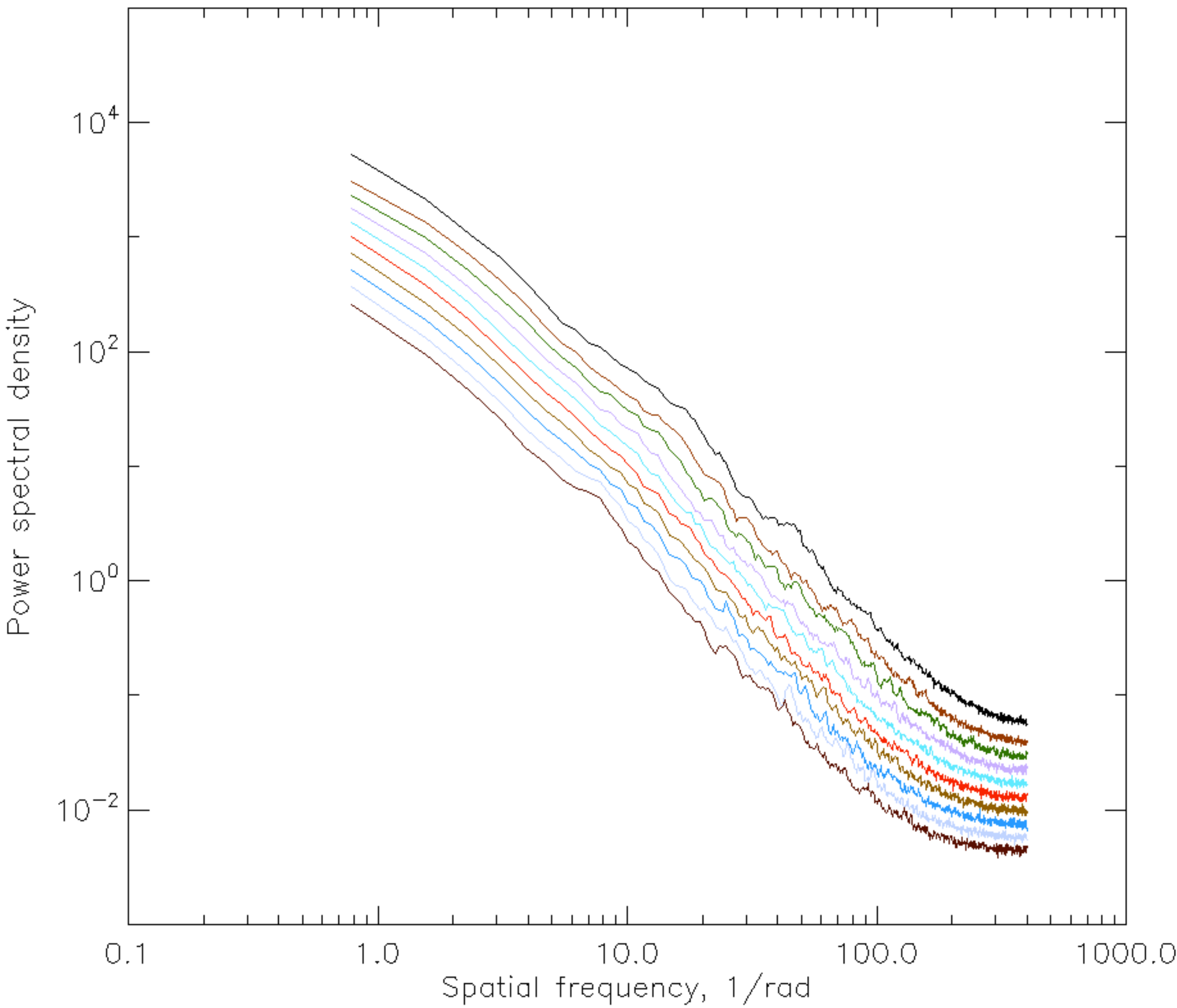


Yearly averages

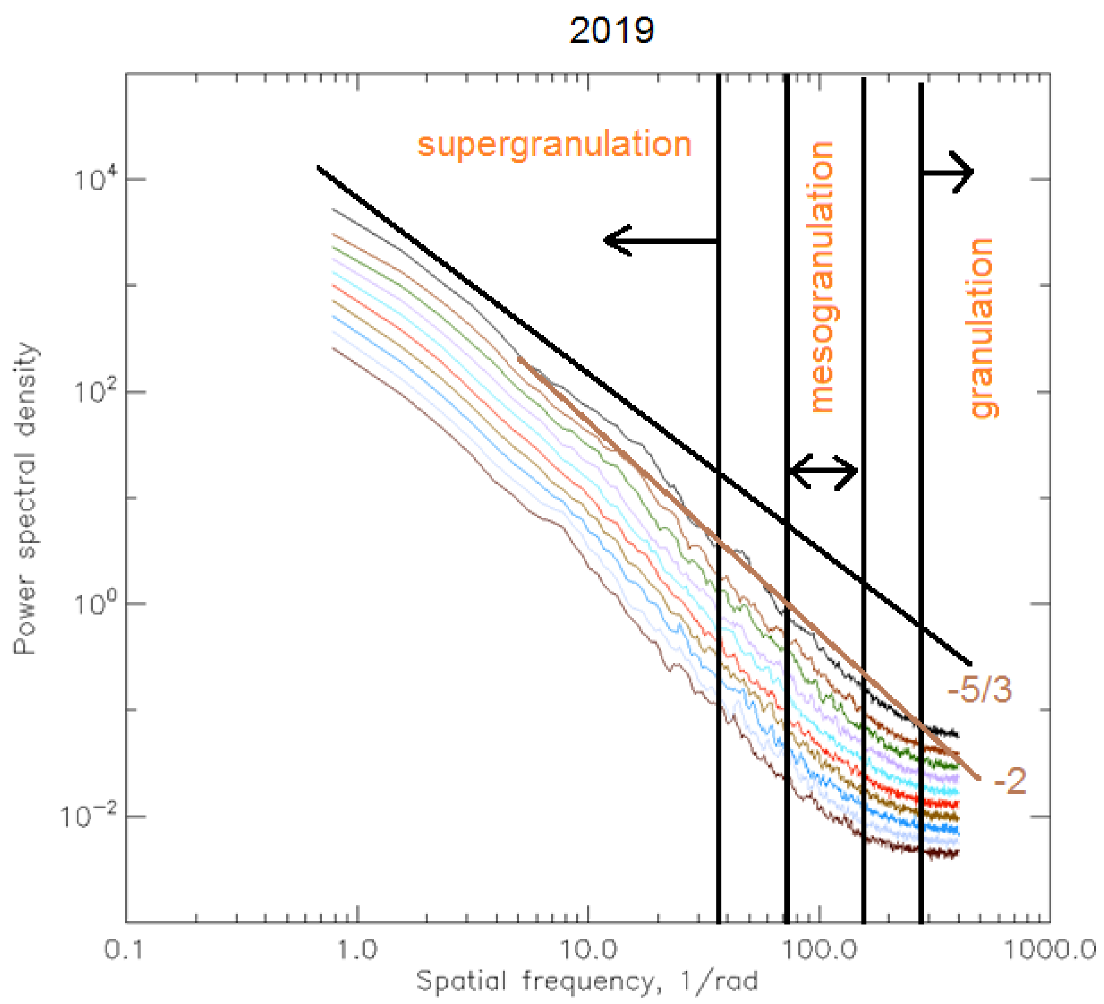
2014, averaged over 1024-point profiles

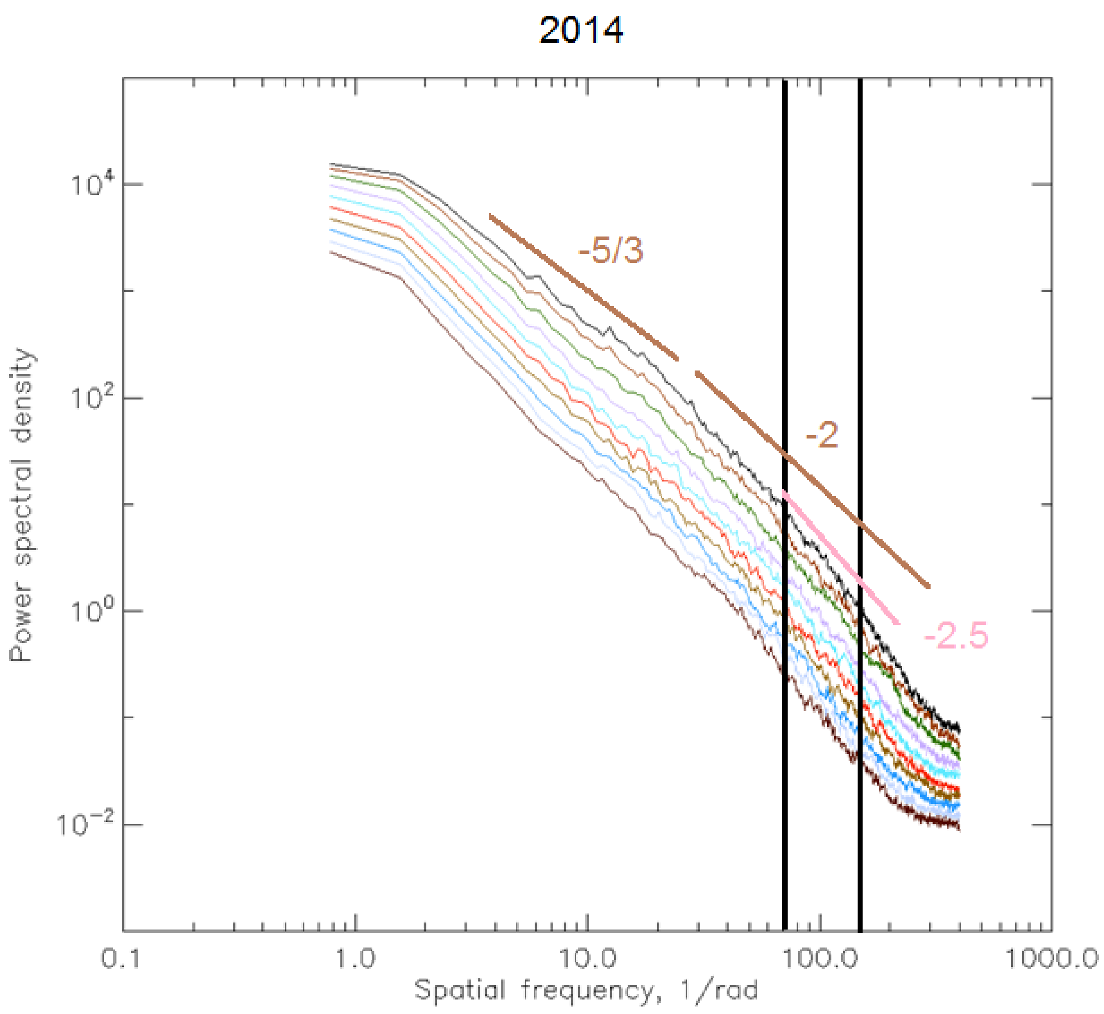


2019, averaged over 1024-point profiles

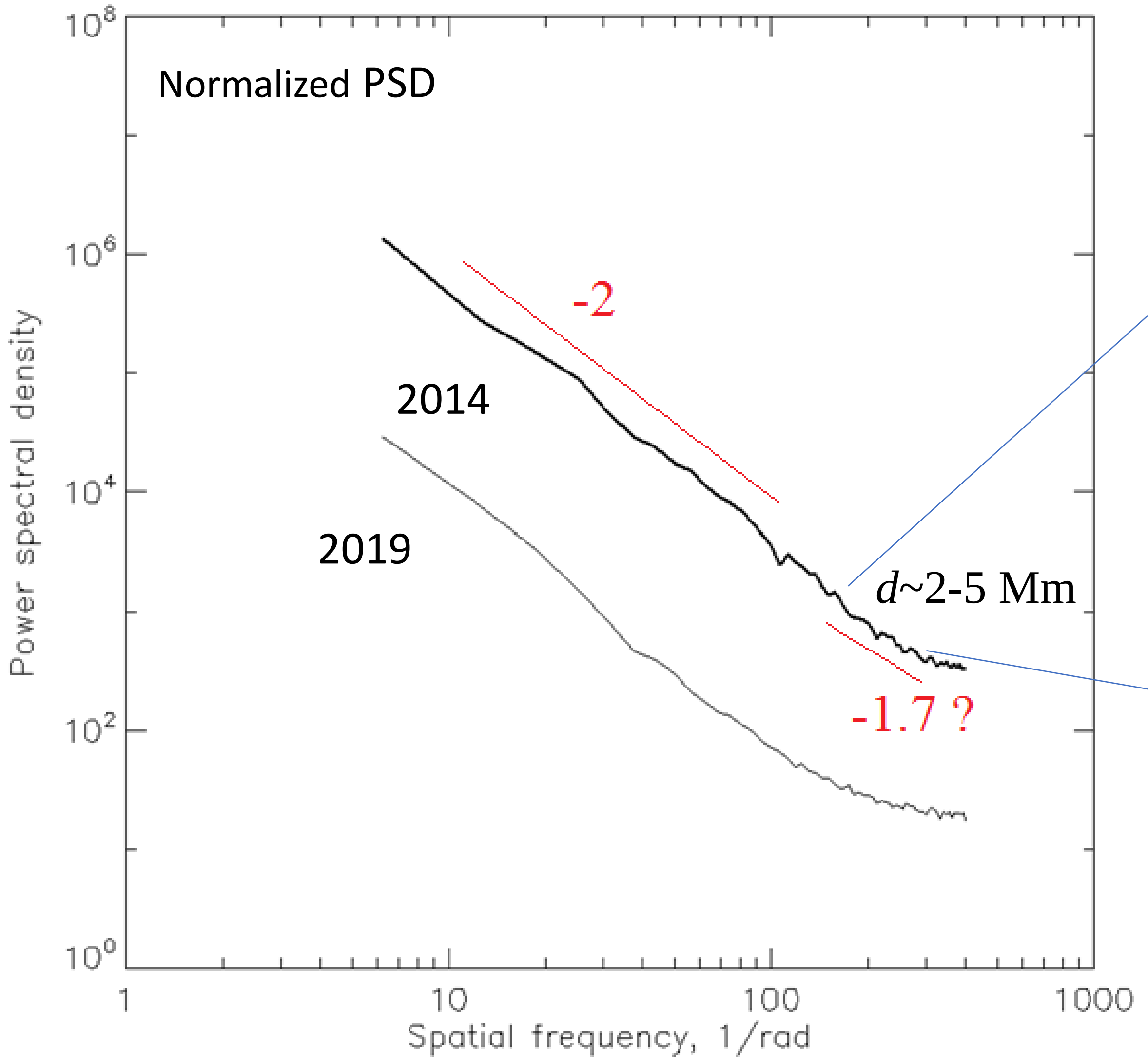


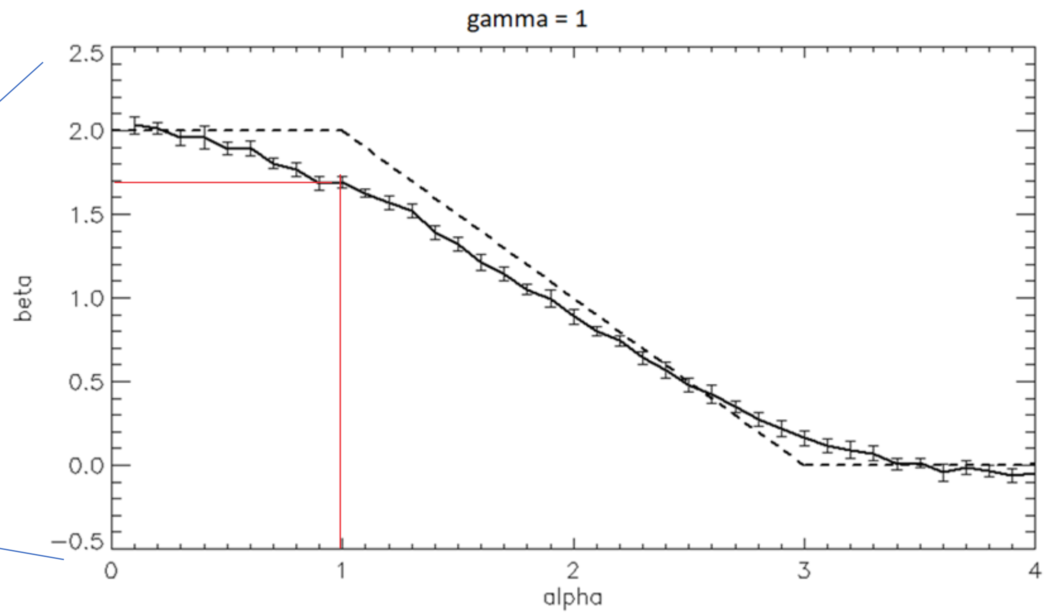
Interpretation (draft)





Assuming power-law scaling of strand sizes / broad distribution of D_s, gamma = 1





α ~ 1 ?

$p(D_s) \sim 1/D_s$

Assuming fixed strand size / narrow distribution of D_s

Relating the constant ("white") low-frequency part of the spectrum with the structure size, assuming fixed D_s.

$$\langle S(k_\perp)\rangle \sim \int p(s)\frac{s^2 ds}{1+k_\perp^2 D_s^2} \sim \int \delta(s-s_0)\frac{s^2 ds}{1+k_\perp^2 D_s^2} \sim \frac{s_0^2}{1+k_\perp^2 D_s^2(s_0)} \tag{19}$$

Using $D_s = cs^\gamma$, this yields

$$\langle S(k_\perp)\rangle \sim \frac{s_0^2}{1+k_\perp^2 c s_0^{2\gamma}} \tag{20}$$

where c is an empirical constant.

In the long-wavelength limit, $k_\perp^2 c s_0^{2\gamma} \sim 0$ and therefore

$$\langle S(k_\perp)\rangle \approx c_1 s_0^2 \tag{21}$$

with c_1 being a proper calibration constant depending on the energy spectrum normalization.

Under certain assumptions, the obtained expression can be used to measure the characteristic length scale D_{s0}. Expressing the net emission impact s_0 of the feature of transverse linear size D_{s0} as

$$s_0 = \int_x^{x+D_{s0}} I_s(x_\perp)dx_\perp \approx D_{s0} < I_{s0} > \tag{22}$$

in which $< I_{s0} >$ is the average emission from the transverse profile of a single feature of size D_s. Consequently we can write

$$D_{s0} \approx \frac{s_0}{< I_{s0} >} = \frac{\langle S(k_\perp \to 0) \rangle^{1/2}}{< I_{s0} >}. \tag{23}$$

Finally, $< I_{s0} >$ can be approximated by the ratio of the average emission signal I_{LOS} to the estimated number n_{LOS} of structures projected onto the same pixel along the light of sight, which results in

$$D_{s0} \approx \frac{\langle S(k_\perp \to 0) \rangle^{1/2}}{< I_{LOS} > /n_{LOS}}. \tag{24}$$

In general, the parameter n_{LOS} should depend on the LOS depth and the filling factor of the structures. Relating n_{LOS} and D_{s0} is necessary for obtaining a close-form solution of the last equation. Alternatively, one can attempt to evaluate $< I_{s0} >$ based on theoretical arguments.

To be continued…