The Thickness of Current Sheets and Implications for Coronal Heating

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Topological Dissipation (Parker 1972)

Continuous 3D magnetic fields (without separatrices or X-points) MUST develop infinitely thin current sheets when subjected to continuous motions at a line-tied boundary.

Infinitely thin current sheet = tangential discontinuity = singular current sheet

Straightforward explanation of coronal heating.

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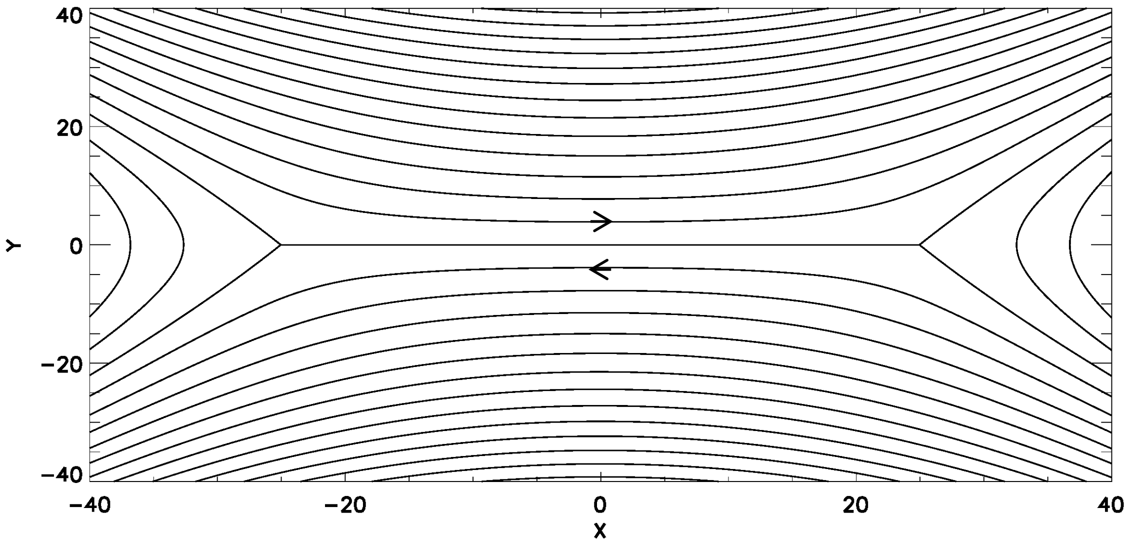
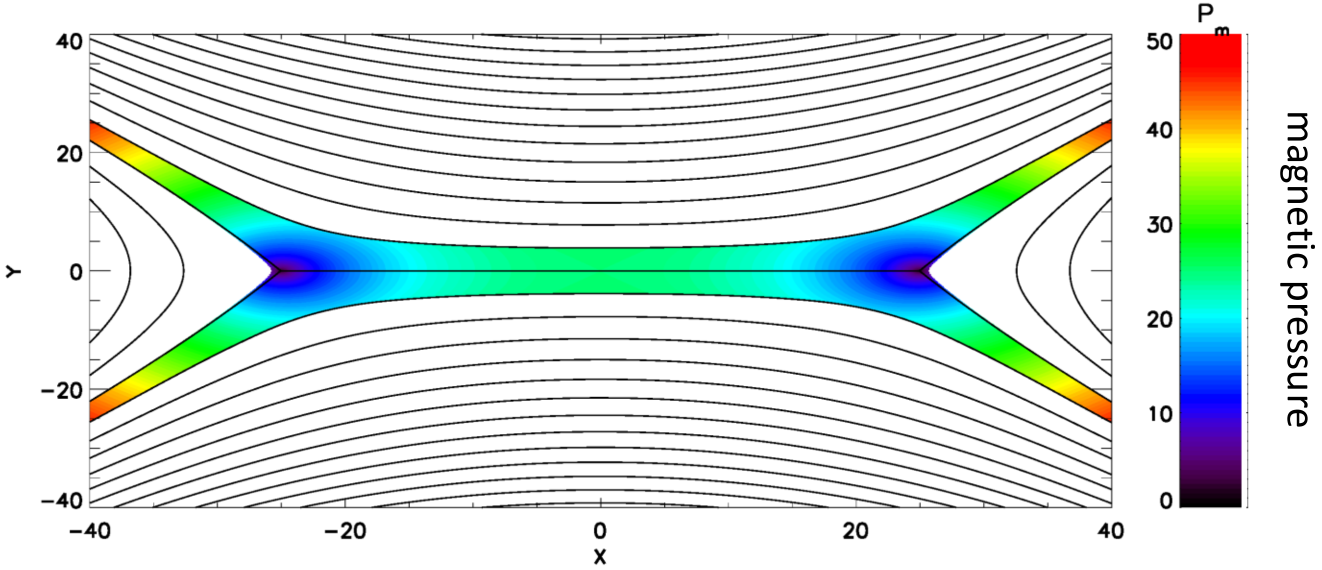
No – infinitely thin sheets are problematic! Magnetic stress and energy cannot build up.

$$F = \frac{1}{4\pi} v_h B_h B_v$$

$$\frac{B_h}{B_v} \approx \tan(10^{o})$$

Infinitely thin current sheet



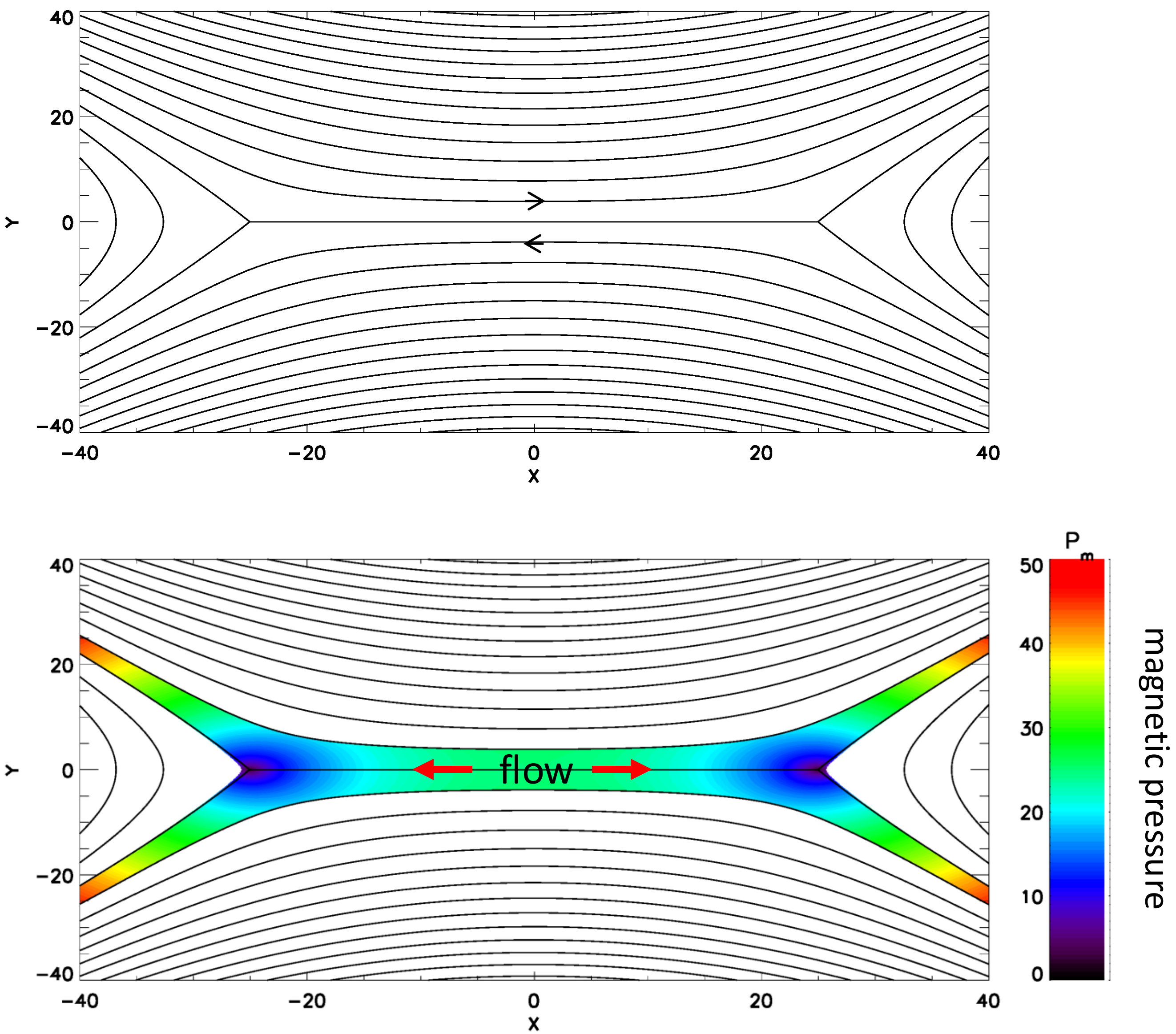


$$\frac{1}{c}(\boldsymbol{J} \times \boldsymbol{B}) = 0$$

$$\nabla\left(\frac{\boldsymbol{B}^2}{8\pi}\right) = \frac{1}{4\pi}(B \cdot \nabla)\boldsymbol{B}$$

Green (1965)

Infinitely thin current sheet



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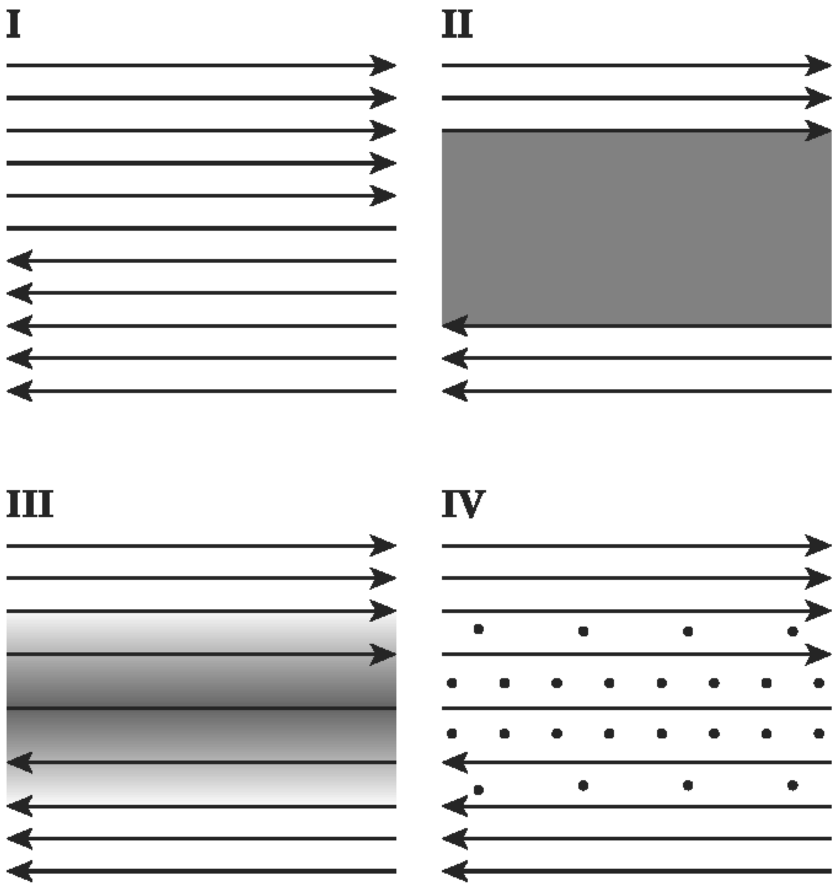
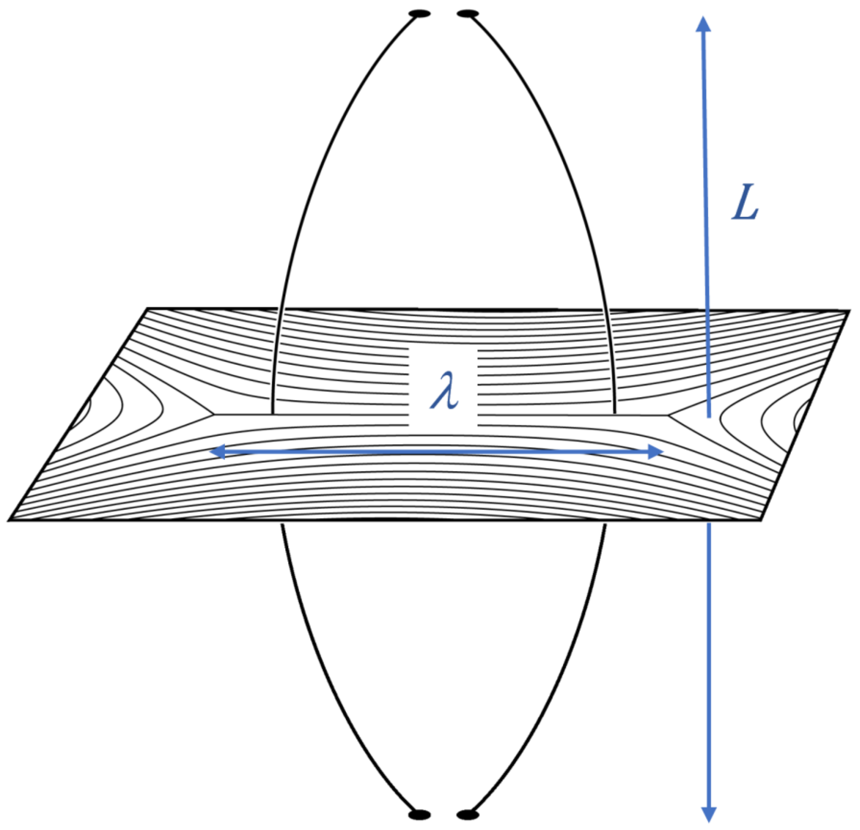
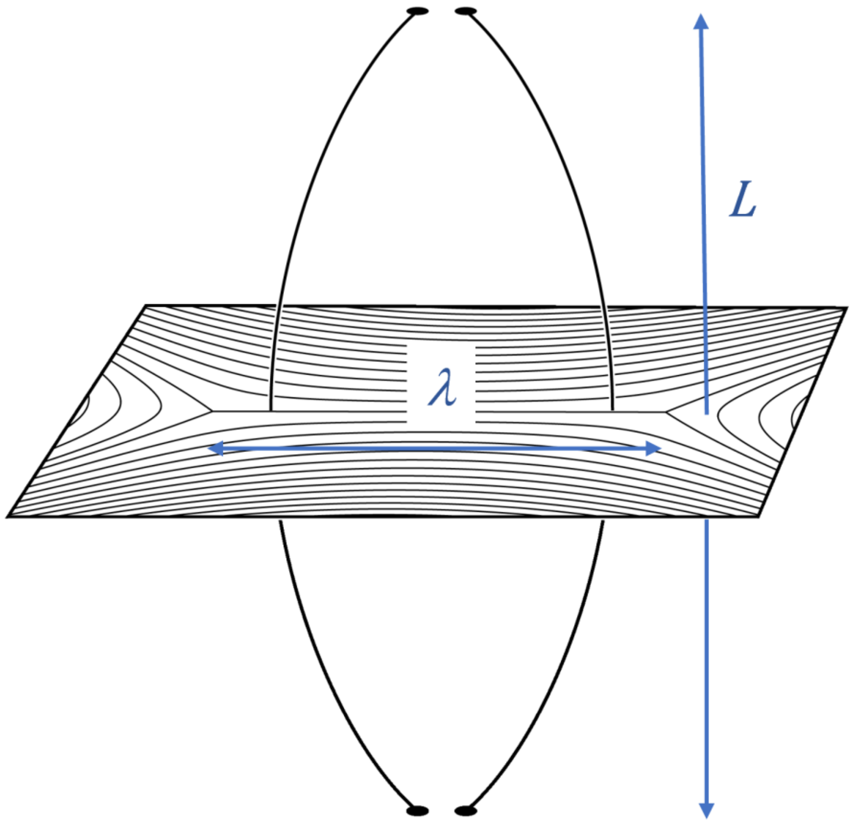

I
II
III
IV



Tension in curved line-tied guide field.

Opposes pressure gradient force along sheet.



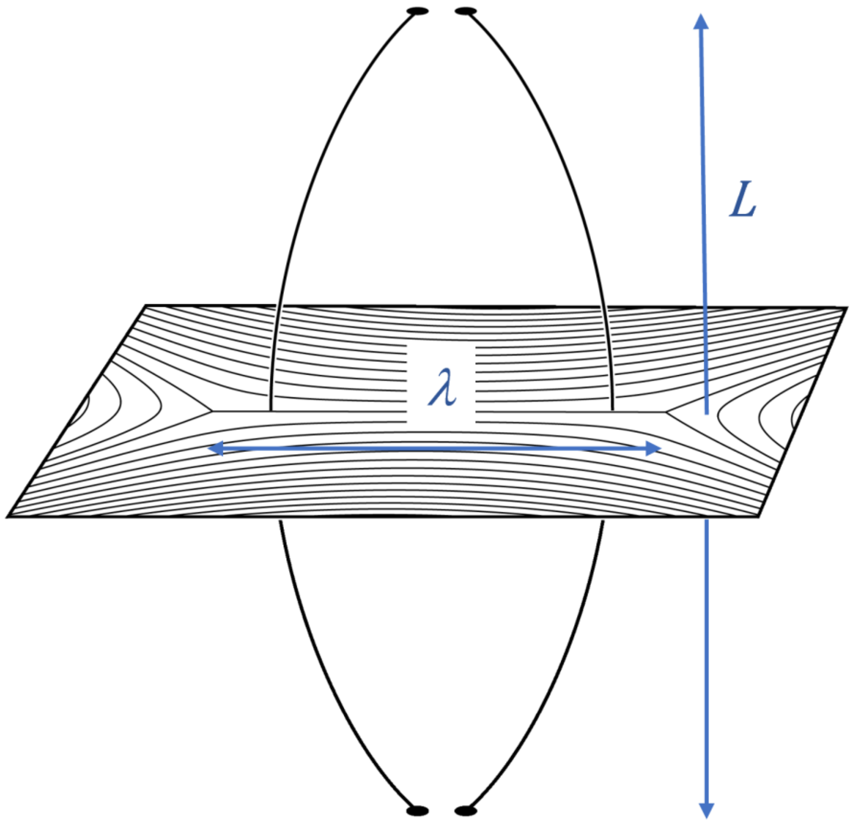
$$\frac{F_t}{F_p} \approx \frac{\lambda}{R_c}\left(\frac{B_{g0}}{B_{x0}}\right)^2$$

$$R_{c,min} \approx \frac{1}{\lambda}\left(\frac{L}{2}\right)^2$$

$$\frac{F_t}{F_p} \leq 4\left(\frac{\lambda}{L}\frac{B_{g0}}{B_{x0}}\right)^2$$

$$\frac{B_{x0}}{B_{g0}} > \left(\frac{B_{x0}}{B_{g0}}\right)_{crit} = \frac{2\lambda}{L}$$

B_{g0}, B_{x0} = guide field and reconnecting field components outside of sheet



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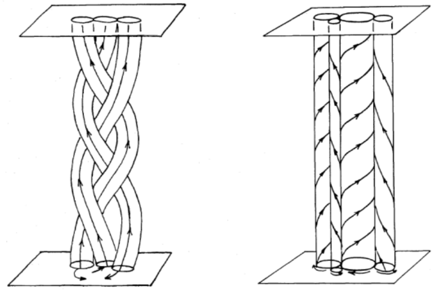

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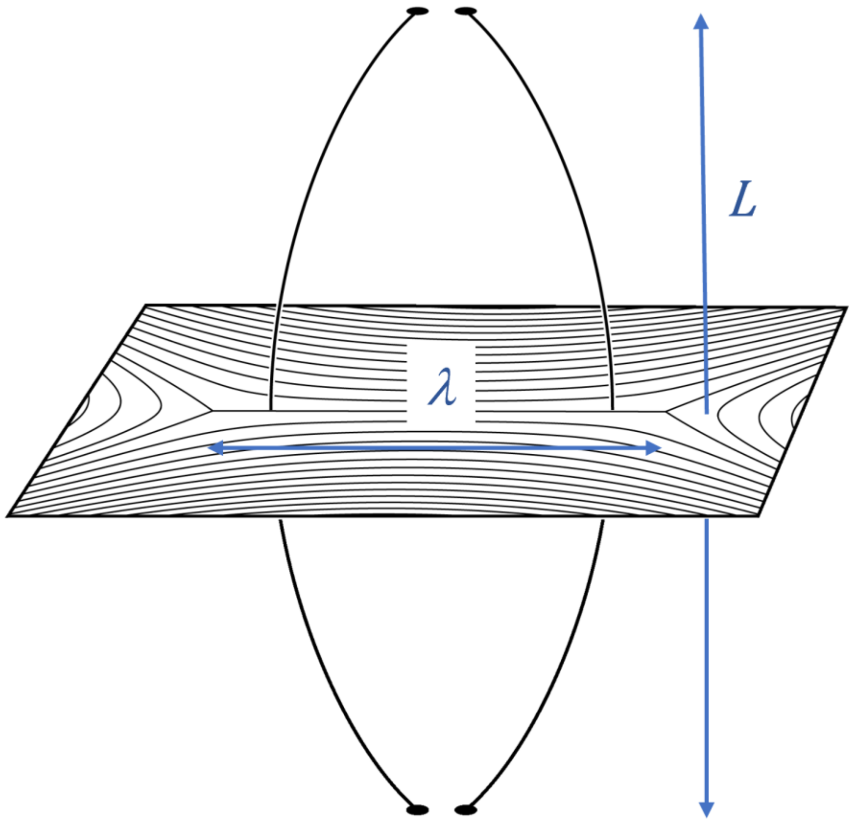


$$\frac{B_{x0}}{B_{g0}} > \left(\frac{B_{x0}}{B_{g0}}\right)_{crit} = \frac{2\lambda}{L}$$

$L = 50{,}000$ km

$\lambda = 2{,}000$ km

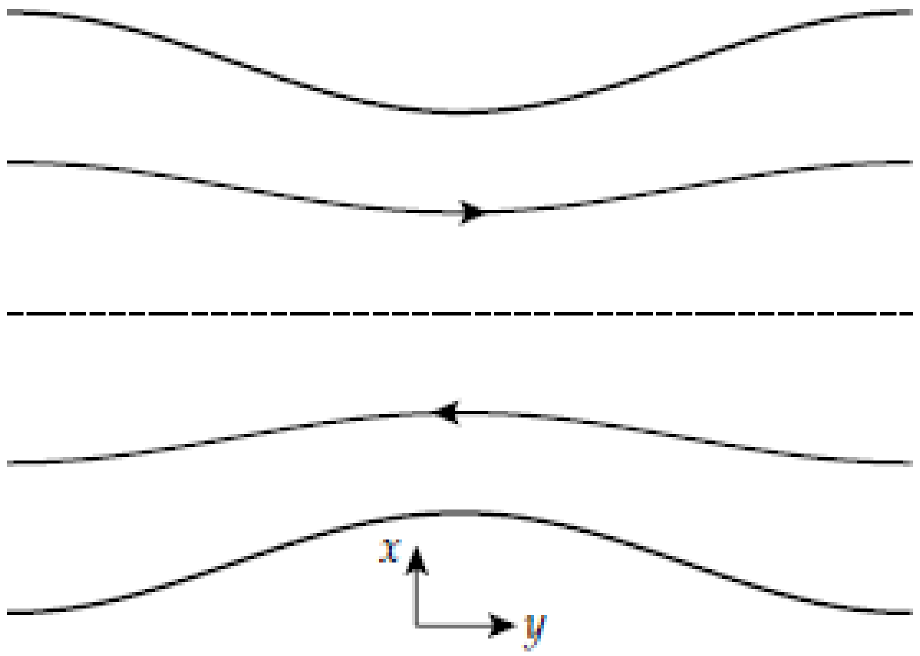
→ $(B_{x0}/B_{g0})_{crit} = 0.08$ (10° shear angle)

Consistent with observed heating requirements in active regions.

Hahm-Kulsrud-Taylor (HKT) Problem

Zhou, Huang, Qin, & Bhattacharjee (2018)

$L_{crit} = 29\ \lambda$

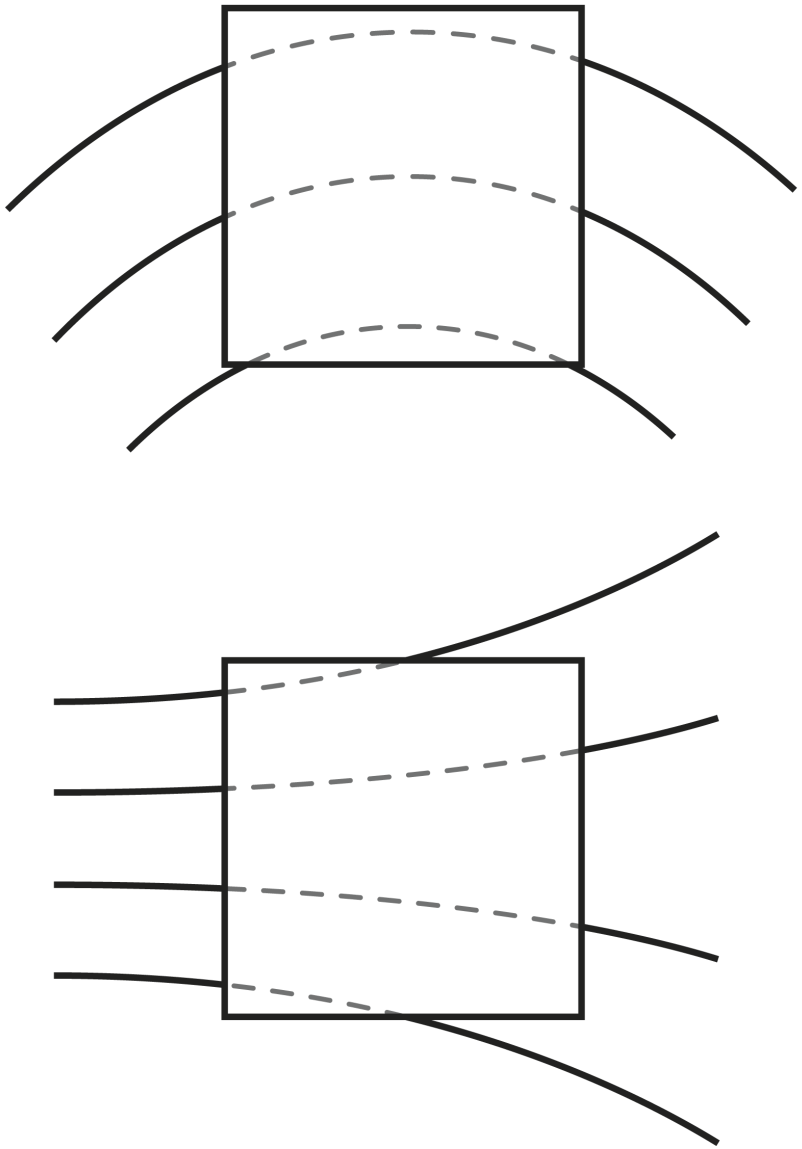


Huang et al. (2022)

Our formula:

$$L > L_{crit} = 2\lambda \left(\frac{B_{g0}}{B_{x0}}\right)$$

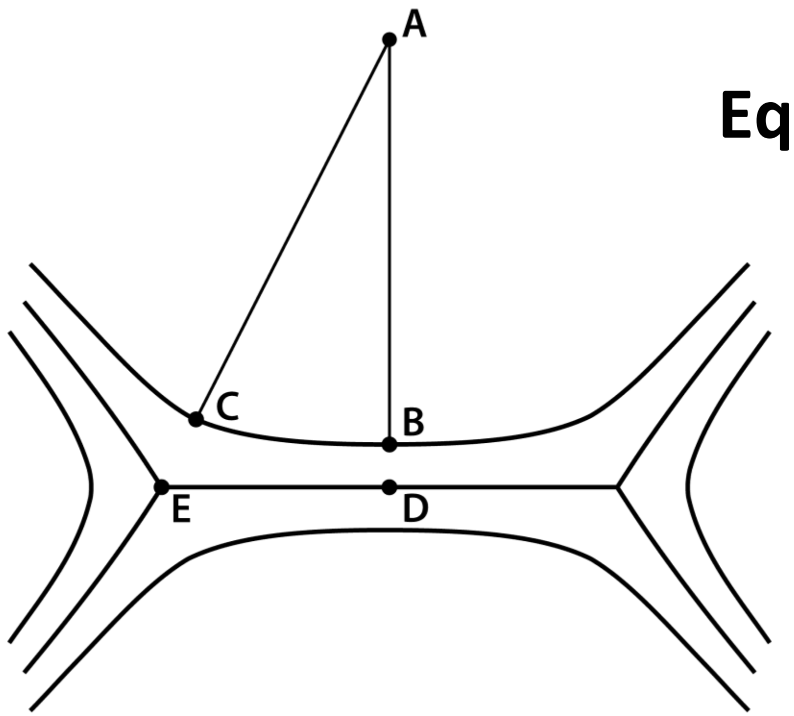
$$= 11\ \lambda$$



$$T_{ij} = \frac{1}{4\pi}\left(B_iB_j - \frac{1}{2}B^2\delta_{ij}\right)$$

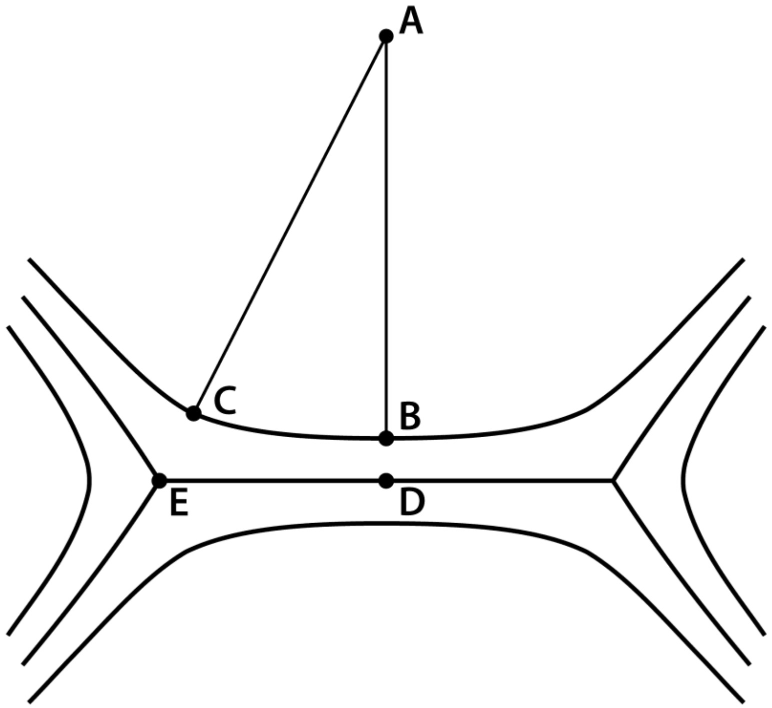
Equilibrium possible?

$B = 0$ in sheet



- Plasma pressure must be constant along field lines (and within entire sheet).
- Total pressure is constant across a magnetic boundary.
- Therefore, $B_C = B_B$.
- But $B_C < B_B$ to satisfy force balance outside the sheet. [tension force = $B^2/(4\pi R_c)$]
- No equilibrium exists at finite thickness (2D and 2.5D)

$B = 0$ at center of sheet
P_{max} at center of sheet



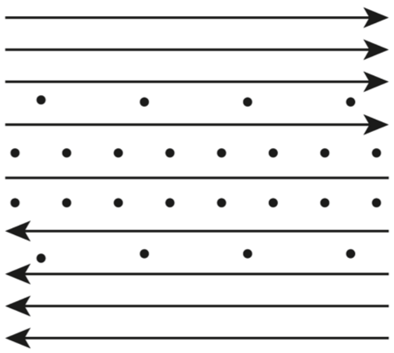
$$P_E < B_C^2/8\pi$$

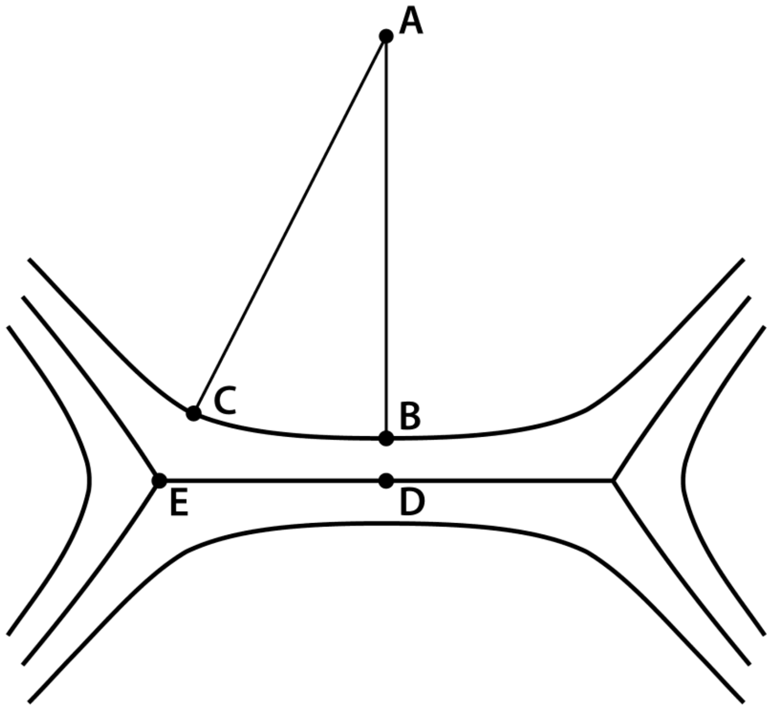
$$B_C^2/8\pi < B_B^2/8\pi$$

$$P_D \approx B_B^2/8\pi$$

$$P_E < P_D$$

But $P_E = P_D$ in equilibrium. Horizontal outflow from center of sheet.





Replace plasma with a straight guide field.

P = guide field pressure, $B_{guide}{}^2/8\pi$

Field rotates by 180° across sheet.

$$P_E < B_C{}^2/8\pi$$

$$B_C{}^2/8\pi < B_B{}^2/8\pi$$

$$P_D \approx B_B{}^2/8\pi$$

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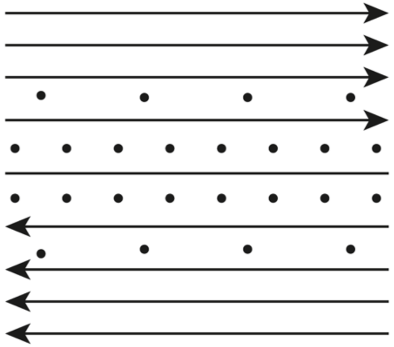

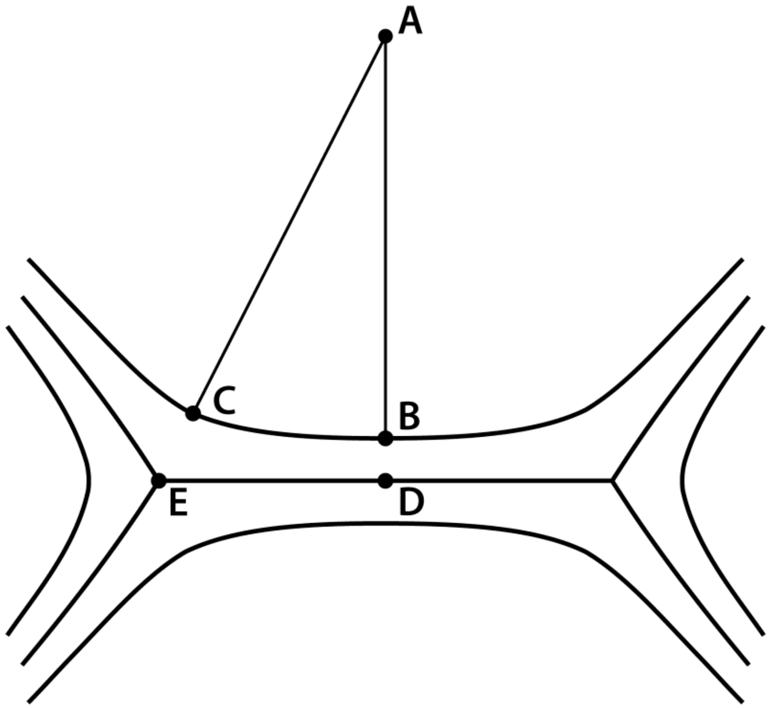

Replace plasma with a straight guide field.

P = guide field pressure, $B_{guide}^2/8\pi$

Can add a uniform guide field, B_{g0}.

Field rotates by < 180°.

$$P_E < B_C^2/8\pi$$

$$B_C^2/8\pi < B_B^2/8\pi$$

$$P_D \approx B_B^2/8\pi$$

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