Planets and Disks

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Theoretical overview

• Gaps, Clearings and sharp Edges in disks, how they are interpreted in terms of planets
• Planet/disk interactions and regimes
• Multiple planet systems and evolution
Gap opening in a gas disk

Spiral density waves are driven at Lindblad resonances

Planet mass ratio $q = \frac{M_p}{M_*}$

Torque on disk from each resonance depends on $q$ and Fourier component of gravitational perturbation (Torque formula)

$$T_m = -f_m q^2 \Sigma r_p^4 \Omega_p^4$$

Torque independent of type of viscosity.

Stronger Lindblad resonances nearer the planet, but the location is shifted due to the sound speed $\rightarrow$ torque cutoff. Add the torques from resonances to estimate total ...
Minimum Gap Opening Planet In an Accretion Disk

Assumption: Torques from waves launched by planet are not overcome by viscous spreading torque

\[ q \gtrsim 40 \text{Re}^{-1} \]

Lin, Crida, Bryden ...

Drop in accretion rate, or self-shadowing

For dense disks, large planets are required to account for disk edges

Self-shadowed regions allow lower mass objects to open gaps

Edgar et al. 07

\[ q_{\text{min}} \propto \dot{M}^{0.48} \alpha^{0.8} M_*^{0.42} L_*^{-0.08} \]
Connections between disk morphology and planet

- Number of arms, slope, clearing time for gap, depend on planet mass.
- For small planets many waves are in phase → single arm (Ogilvie)
- Higher planet masses leave a beat frequency between 2-3-4 armed waves

**Fig. 1** Surface density perturbation for a 4 $M_\odot$ planet (located at $(x,y) = (-1,0)$) embedded in a disc with $h = 0.05$, showing the prominent spiral wakes associated with Lindblad torques.
Caveats/Complications

- Turbulence (e.g., Pardekooper, Ketchum)
- Multiple planet interactions
- Multiple planet scattering
- Resonance trapping
- Dead-zones
- Illumination
- Planet traps
- Planet formation+growth

Moekel & Armitage ‘11
Gap opening in a dusty debris disk

• Taking the limit of low disk opacity
  Are spiral density waves driven at Lindblad Resonances?

To answer this question first consider general properties of resonances
Resonances

• Regions where small perturbations add up and no circular orbits exist

• When do perturbations become constructive?
Resonant angle

\[ jn_p - (j + 1)n = 0 \]

mean motions are integer multiples

\[ \phi = j\lambda_p - (j + 1)\lambda = \text{constant like 0 or } \pi \]

**Resonant angle** remains fixed

Librating resonant angle \(\leftrightarrow\) in resonance

Oscillating resonant angle \(\leftrightarrow\) outside resonance
Resonant Angle

\[ j\lambda_p - (j + 1)\lambda - \omega = 0, \pi \]

\[ j(\lambda_p - \lambda) + \{0, \pi\} = \lambda - \omega = M \]

Mean anomaly which is zero at pericenter.
Location in orbit radial oscillation related to angle on sky w.r.t. planet.
Dimensional Analysis on the Pendulum

• $H$ units
• Action variable $p$
• $a$ $\text{cm}^{-2}$
• $b$ $\text{s}^{-1}$
• Drift rate $\frac{db}{dt}$ $\text{s}^{-2}$
• $\varepsilon$ $\text{cm}^2 \text{s}^{-2}$

Ignoring the distance from resonance we only have two parameters, $a, \varepsilon$

• Only one way to combine to get momentum $\sqrt{\varepsilon / a}$
• Only one way to combine to get time $\frac{1}{\sqrt{a \varepsilon}}$

$H(p, \theta) = a \frac{p^2}{2} + bp + \varepsilon \cos \theta$ $\text{cm}^2 \text{s}^{-2}$ $\text{cm}^2 \text{s}^{-1}$

$\text{(H}=l\omega) \text{ and } \omega \text{ with } 1/\text{s}$
Resonant width and Libration period

Resonant width solve

\[ H(p, \theta) = a \frac{p^2}{2} + \epsilon \cos \theta \]

- Resonant width solve
  \[ H(p, \theta) = 0 \] for maximum \( p \). Distance to separatrix
  \[ \Delta p = \sqrt{2\epsilon/a} \]

- Libration timescale, expand about fixed point
  \[ \ddot{\theta} = -\epsilon a \theta \]

Libration period

\[ \frac{2\pi}{\sqrt{a \epsilon}} \]

note similarity between these expressions and those derived via dimensional analysis
Dimensional analysis on the Andoyer Hamiltonian –
Low e expansion for mean motion or /and Lindblad resonances

\[ H(p, \phi) = ap^2 + bp + \epsilon p^{k/2} \cos(k\phi) \]

- We only have two important parameters. (b sets if distance to resonance)
  \( a \) dimension cm\(^{-2} \) \( \epsilon \) dimension cm\(^{2-k} \) s\(^{-2-k/2} \)

- Only one way to form a timescale and one way to make a momentum sizescale.

\[ \tau = (a\epsilon)^{2/3} \quad \text{for } k=1 \]

- To order of magnitude for j-th first order resonance
  \( \epsilon \sim \mu \delta^{-1} \exp(-j\delta) \quad a \sim j^2 \)
Physical relevance

- $\tau^{-1}$ gives the libration timescale –relevant for
  - driving spiral density waves
  - proximity to resonance, sizes of kicks needed to push system in or out of resonance
- $\tau^{-2}$ rate of change of a frequency
  - The adiabatic limit (relevant for capture)
- The momentum sizescale –relevant for
  - sizes of motions in resonance
  - critical eccentricity ensuring capture in adiabatic limit
  - size of eccentricity jump if fail to capture and jump across the resonance
Collision timescale and driving of spiral density waves

• Spiral density waves are not driven if the libration time of the Lindblad resonance is shorter than time between collisions.

Daphnis edge shadow

At low disk opacity spiral density waves are ineffective at pushing away a disk (e.g. Quillen 2005 in the context of the prediction of Fomalhaut B).
Alternative boundaries for disk edges

• Mean motion resonances become stronger and denser closer to planet. They overlap at $da \sim a \mu^{2/7}$

• $\mu^{2/7}$ law (Wisdom 1980) for the Width of chaotic zone

• Change in dynamics at this boundary

• The edge is potentially sharp.

• Proposed as a relevant boundary for the Fomalhaut disk edge (Quillen 2006)
Chaotic zone boundary and removal time within

$$\frac{\partial}{\partial a} \left( D \frac{\partial N}{\partial a} \right) = \frac{N}{t_{\text{removal}}}$$

What mass planet will clear out objects inside the chaos zone fast enough that collisions will not fill it in?

$$M_p > \text{Neptune}$$
One the prediction of Fomalhaut B in 2006
A single planet assumed to truncate both disk
and account for eccentricity of dust belt
cleared out by perturbations from the planet
$M_p > \text{Neptune}$

Assume that the edge of the ring is the boundary of the chaotic zone. Planet can’t be too massive otherwise the edge of the ring would thicken or show structure $\Rightarrow M_p < \text{Saturn}$
nearly closed orbits due to collisions
eccentricity of ring equal to that of the planet
Diffusive particle disk next to a planet

To truncate a disk a planet must have mass above

\[
\log_{10} \mu > -6 + 0.43 \log_{10} \left( \frac{\tau_n}{5 \times 10^{-3}} \right) + 1.95 \left( \frac{u/v_K}{0.07} \right)
\]

low mass planets can open gaps in cold diffusive disks with long collision timescales
Can the boundary be further away?

Why isn’t Fom B exactly where it was predicted to be?

• Larger mass planet (Chiang et al)
• Excursions due to secular oscillations?
• Planet-planet scattering (e.g., Raymond, Moro-martin)
• Planet eccentricity may not affect chaotic zone boundary (Quillen & Faber 07) though particle eccentricity and pericenter distributions do (Mustill & Wyatt 11)
When can a massive planet be present in a disk and there is no gap?
When can a massive planet be present in a disk and there is no gap?

1) Planet-planet Scattering into a gas disk, on short timescales →

2) Planet-planet scattering into a debris disk, short timescales ←

Moekel & Armitage

Raymond et al.
When can a massive planet be present in a disk and there is no gap?

3) Swift or runaway migration into a debris disk that is sufficiently massive to maintain migration (as discussed by Gomes, Duncan et al.)

These settings can be classified.
Either short timescale or
The disk is dense compared to the planet mass
→ Observable consequences such as dust production
When can you open a gap without a planet?
When can you open a gap without a planet

Fig. 11.— An optical depth profile (top) and true-color image (bottom) of Saturn’s main ring system. Figure from Cuzzi et al. (2010)

Required: cold or low velocity dispersion dense disks
Note gaps are not necessarily empty, and depth is potentially observable.
Will we see exo-disks in a cold stage?
Circum-planet and Circum-secondary Disk Eclipses

Disk eclipses are predicted to be numerous!

Only a few known now.....

- Eric Mamajek, Mark Pecaut (discovers of J1407 event)
- Erin Scott, Fred Moolekamp (eclipse modeling)
- Matt Kenworthy, Andrew Collier Cameron & Neil Parley
J1407 Nightly averages

Gaps
Rings

ASAS
SWASP

J1407

days

HJD - 2450000
Eclipse models

Fine structure in eclipse $\rightarrow$ a very thin disk with lots of structure, like Saturn’s rings
disk aspect ratio of $h/r \sim 0.01$

Impact parameter leads to asymmetry in eclipse curve
Non unique eclipse model by Erin and Fred!

If gaps in light curve interpreted in terms of gap opening objects then their mass is of order $10^{-3}$ of secondary
Gap opening objects likely pretty small

$$\frac{t_{gap}}{t_{eclipse}} \sim \left( \frac{m_{\text{satellite}}}{3m_2} \right)^{\frac{1}{3}}$$
Forming planetary systems seen in eclipse

• J1407 star is 16Myr, solar type star.
• Forming planetary system, seen in occultation not reflected or thermal radiation
• potentially new views of forming planetary systems
• (+ eclipsing systems should be common)
Cold disk Components

May help explain

• Spirals and arcs (e.g. HD100546, HD141569, GM Aur, and other disks presented here)
• Clumps in some disks (e.g. AU Mic ...)
• Substructure in other debris disks
When do you get interesting structures due to resonances?

- Resonance capture scenarios
- Liou & Zook, Ozernoy et al., Kuchner & Holman early predictions of structure
Another model

Adam Deller and Sarah Maddison’s resonant capture model account for disk eccentricity but not sharp edge collisions ignored.
Structures generated by a migrating planet

Reche et al 08

Kucher & Holman 03
Morphology of diffusive disks near planets

- Featureless for low mass planets, high collision rates and velocity dispersions
- Particles removed at resonances in cold, diffuse disks near massive planets
Predicting morphology

• Capture probability can be analytically predicted for all resonances either for dust spiraling via PR drag or due to migration of a planet. Identification of non-adiabatic limit, sensitivity to eccentricity and subresonances (Quillen 2006). Recent applications (Mustill & Wyatt 2011). Low eccentricity expansion is justified! (how many situations are we that lucky?)

• Using similar formalism one can estimate when turbulence or close encounters knock things out of resonance

• However: Resonant structure is smoothed by high eccentricity resonant population and estimating lifetimes in resonances is difficult.
How many planets are needed to account for clearings?
How many planets are needed to account for clearings?

• 0 - clearings are due to photoevaporation

• 1 – a single high eccentricity planet is all you need (+ Kozai resonance or previous scattering event)

• Many – Stability timescales leveraged to estimate the number of rocky/icy/gas giants in circular orbits needed to clear the system during its lifetime.

Note: clearings are not necessarily empty → observables hopefully differentiate between these possibilities
Integrated Orbit Crossing timescales for the densely packed Uranian satellite system

Time of first orbit crossing (each pair of moons shown) as a function of mass scaling factor. Power-law relation first seen in integrations by Martin Duncan & Lissauer 97 and John Chambers et al. 96
Disk Clearing by Planets

\[ \log(t_c) = A + B \log(\mu/10^{-7}) + C(\delta/\mu^{1/4}) \]

Relationship between spacing, clearing time and planet mass

Invert this to find the spacing, using age of star to set the stability time.

Stable planetary system and unstable planetesimal ones.

Faber & Quillen 07, Power law for stability based on Chambers et al.96
Closely packed non-resonant Multiple planet systems

How close can we pack planets and still have a stable system?

MMSN = minimum mass solar nebula

Closely packed gas giants require massive disk above MMSN

MMSN with ices only forming closely spaced Neptunes

MMSN with rocky component only, forming closely spaced Earths
Signatures of Three-body resonances in integrations

Characteristic related motions of three semi-major axes for consecutive bodies.

Frozen Laplace angles involving three consecutive bodies.
Source of Instability in closely packed systems

- Large number of three-body resonances allows continuous wander in closely spaced systems.
- Source of longer term instability
- Resonance overlap criterion
Islands of stability

- HR8799 system has closely spaced massive planets in presence of debris disk is likely unstable
- 1:2:4 resonance (Gozdziewski & Migaszewski, Fabricky & Murray-Clay) is a tiny island of stability

Even small motions in outer planet due to interaction with a disk can vastly decrease or increase time till first planet crossing event

See Alex Moore for the latest on this
Circumplanetary disks

Fomalhaut B’s unusual color has led to some interesting speculation

1) circumJovian accretion disk – e.g., Canup & Ward – possibly long lived at large radius
2) Dust swarm due to irregular satellites (e.g. Bottke et al. 2010, Kennedy & Wyatt 2010)
3) Capture of irregular satellites is particularly easy around an outer slowly migrating planet (Hasan et al.)
That was then / this is now

• A few years ago, the proposal that a feature in a disk was caused by a planet was considered a wild possibility. (e.g., will anybody believe it if I propose a Neptune mass object at 119AU?)
• Now multiple planet/disk interactions and scenarios receiving broad spectrum of study