Physics of Planetary Rings
and Signposts of Planets

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INTRODUCTION

1655: Huygens says that Saturn « is encircled by a ring, thin, plane, nowhere attached, inclined to the ecliptic.»

1675: Cassini discovers the Cassini division, and suggests that the rings are made of boulders orbiting Saturn, hitting each other.
INTRODUCTION: TIDES

Maxwell shows it can only be so, because of tides.

Saturn's tides > self-gravity: the rings are inside the Roche limit.

Reminder: Tidal forces (per mass unit):

\[ \Omega = \left( \frac{GM}{r^3} \right)^{1/2} \]

\[ F_g = \frac{GM}{(r+/-a)^2} \]

\[ F_c = \Omega^2(r+/-a) \]

\[ F_{\text{tides}} = 3\Omega^2a \]
**INTRODUCTION: TIDES**

Self gravity force (per mass unit) : 
\[ F_{sg} = Gx(4/3)\pi a^3 \rho / (2a)^2 \]

Stability condition for the aggregate : 
\[ F_{sg} > F_{tides} \]

or : 
\[ r > (9M/\pi \rho)^{1/3} = r_{Roche} \]

Application:

\[ M = M_{Saturn}, \]
\[ \rho = 600 \text{ kg.m}^{-3} \]
\[ r_{Roche} = 1.4 \times 10^8 \text{ m} \]
\[ r_{outer\ edge} = 1.36 \times 10^8 \text{ m} \]
In the rings, accretion and tidal disruption of aggregates are the norm.

Movie by Hanno Rein, in a shearing box, in the frame corotating with the blue spot.
Composition:

10cm-10m blocks (French & Nicholson 2000, Cuzzi et al. 2007) of more than 90% pure water ice (eg Cuzzi et al. 2010), on circular, coplanar orbits: \( e \sim 0 \), \( i \sim 0 \) (\( H/r = 10^{-7} \)).

**Saturn's main rings are dynamically cold, dominated by tides.**
DYNAMICS: the E-ring

But the E-ring is beyond $r_{\text{Roche}}$, and thick.
DYNAMICS: the E-ring

Comes from Enceladus geysers!
SIGNPOSTS

Trajectories of ring particles perturbed by a moonlet, in the frame corotating with the moonlet (Tiscareno et al. 2008)
SIGNPOSTS of MOONLETS

Gaps are open by a small satellite (Cuzzi & Scargle 1985). Where is it?
SIGNPOSTS of MOONLETS

Pan inside the Enke gap:

SIGNPOSTS of MOONLETS

Zoom on the Enke gap:
horseshoe region + waves associated to Pan.
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Daphnis, as it scatters ring particles, opening the Keeler gap:
SIGNPOSTS of MOONLETS

Daphnis, as it scatters ring particles, opening the Keeler gap:
SHEPHERDING by MOONLETS

F ring: repelled outwards by Prometheus, inwards by Pandora.
SIGNPOSTS: PROPELLERS

Tiscareno et al. (2006, 2008), Sremcevic et al. (2007)
SIGNPOSTS: PROPELLERS

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Trajectory integrated with Bulirsch-Stoer algorithm (Crida et al. 2010)
The difference between inner and outer trajectory can be measured: $\delta$ a.m. Integrated with Bulirsch-Stoer algorithm, to numerical precision. (Crida et al. 2010)
Result:

Torque on the planet = \( \sim - 18 \left( \frac{\Sigma}{M_{\text{Sat}} r_p^{-2}} \right) \left( \frac{M_p}{M_{\text{Sat}}} \right)^{4/3} M_{\text{Sat}} r_p^2 \Omega_p^2 \)

Not type I migration.

Not enough to account for observations.

(Crida et al. 2020)
SIGNPOSTS: PROPELLERS

But the ring particles clump (Colwell et al. 2006, Hedman et al. 2007). Random torques, and random walk (Rein & Papaloizou 2010).

- Explains the observed direction reversal of the migration.

- Compatible amplitude (depending on that of the clumping).
Mimas' 2:1 MMR marks the outer edge of the B-ring.

This edge has an m=2 shape (Nicholson et al 2009, 2011).
Janus' 7:6 MMR marks the outer edge of the A-ring.

This edge has an m=7 shape.
Many other resonances make waves. Gravity supported, propagate towards the satellite.
Every astrophysical disk in Keplerian rotation spreads, because of viscous stress (see Lynden-Bell & Pringle 1974).

The inner disk rotates faster, so the friction accelerates the outer disk, and slows down the inner disk (that falls onto the central body).
RINGS EVOLUTION

Mass conservation:
\[ r \frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial r} \left( r \Sigma v_r \right) = \cdot \]

Angular Momentum Conservation:
\[ \frac{\partial}{\partial t} \left( \Sigma r^2 \Omega \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( v \Sigma r^3 \frac{\partial \Omega}{\partial r} \right) = 0 \]

Density evolution:
\[ \frac{\partial \Sigma}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} \left[ \sqrt{r} \frac{\partial}{\partial r} \left( v \Sigma \sqrt{r} \right) \right] \]
RINGS EVOLUTION

Viscosity in self-gravitation rings (Daisaka et al. 2001):

\[
\nu = \nu_{\text{coll}} + \nu_{\text{trans}} + \nu_{\text{grav}}.
\]

Where \( \nu_{\text{trans}} \) is the translational viscosity, the advection of a.m. due to the random motion of particles,

\( \nu_{\text{coll}} = a^2 \Omega \tau \) is due to collisions (\( \tau \)=opacity),

\( \nu_{\text{grav}} = 0 \) if \( Q>2 \), \( \nu_{\text{grav}} = \nu_{\text{trans}} \) if \( Q<2 \).

Parameter \( Q \) of Toomre: \( Q = \Omega \sigma_r / (3.36 \, G \, \Sigma) \).

where \( \sigma_r = \nu_r \) dispersion.
The spreading slows down as soon as $Q>2$.

Convergence towards the $Q=2$ profile. (Salmon et al. 2010)
Whatever the initial mass, the mass of the rings after 4.5 Gyr is about the present mass (Salmon et al. 2010).

What happens to the lost material?
Outside the Roche limit, it aggregates and forms new satellites. They migrate away from the rings.

Janus, Epimetheus, Pandora, Prometheus, Atlas, are:
- under dense (~600 kg.m\(^{-3}\))
- same spectrum as the rings
- dynamically young
- young surfaces

They formed this way (Charnoz et al 2010).
Numerical simulation of ring spreading with $r_{\text{Roche}} = 140$ thousand km.
Beyond? There could have been enough mass in the rings initially to form all satellites until Rhea (Canup 2010).
Beyond Mimas, no more MMR, no more migration?

**Solution:**

Tides from Saturn are enough if \( Q_{\text{Saturn}} = 1680 \), as suggested by Lainey et al.

« Tidal ages » of satellites are:
- smaller than 3.5 Gyrs.
- an increasing function of the orbital radius

\[
\frac{da}{dt} = \frac{3 k_{2p} M_{\text{satellite}} \sqrt{G R_{\text{Saturn}}^5}}{Q_{\text{Saturn}} \sqrt{M_{\text{Saturn}} a^{11/2}}}
\]
Charnoz et al. 2011:

Initially, the rings were very massive, with a small fraction of silicates. Silicates are dense -> they accrete and form chunk, coated with ice, which migrate.

Some of them end outside the rings, and keep migrating outwards, while accreting ice from the rings. This makes a series of differentiated satellites of random silicate/ice ratio, and non spherical cores.

It also explains the mass-distance distribution (Crida & Charnoz 2011 DPS).
Rings and satellites / moonlets are intimately linked:

- Rings are perturbed by the satellites (waves, MMR, propellers, gaps),
- in turn, rings perturb satellites orbits (migration),
- the E-ring of Saturn originates from Enceladus geysers
- many satellites of Saturn are born from the rings.
Satellites enfants des anneaux